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AN EVALUATION OF THREE RELIABILITY GROWTH MODELS

Richard Oren Neal

NAVAL POSTGRADUATE SCHOOL

Monterey, California

THESIS

AN EVALUATION OF THREE RELIABILITY GROWTH MODELS

by

Richard Oren Neal

June 1978

Thesis Advisor: W. Max Woods

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This thesis presents an evaluation of three relatively simple reliability growth models for which accuracy, precision, and robustness performance were examined over a wide variety of true underlying reliability growth patterns. A continuous cumulative failure rate model, a continuous instantaneous failure rate model, and a discrete reliability model, each of which employ ordinary regression methods, were evaluated using standard computer Monte Carlo simulation techniques. Simple, straightforward statistical measures of performance

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An Evaluation of Three Reliability Growth Models

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Submitted in partial fulfillment of the
requirements for the degree of

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June 1978

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This thesis presents an evaluation of three relatively simple reliability growth models for which accuracy, precision, and robustness performance were examined over a wide variety of true underlying reliability growth patterns. A continuous cumulative failure rate model, a continuous instantaneous failure rate model, and a discrete reliability model, each of which employ ordinary regression methods, were evaluated using standard computer Monte Carlo simulation techniques. Simple, straightforward statistical measures of performance are exhibited in graphical and tabular form. All the models displayed some degree of difficulty in tracking particular types or portions of anomalous reliability growth patterns. The cumulative model displayed this difficulty the least and exhibited good variability (precision) performance providing confidence in its use. The instantaneous model, while displaying generally good accuracy, exhibited poor variability performance. Except for a couple of anomalous situations, the discrete model showed good accuracy and variability performance. Forecasting performance of all the models proved to be worse than their capability to determine current reliability status.

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I. INTRODUCTION

New weapons systems components characteristically display very low initial reliability as they begin progress through early development and testing. This low infant reliability of a system and its components is common to many commercial systems such as automobiles, aircraft, etc. as well as complex military systems. The reticence of contractors to divulge early test data that reveals this low infant reliability can handicap the project officer and contractor in exercising their managerial responsibility to bring the system in on time at the specified reliability level. Because reliability growth is not tracked in early development stages, timely opportunities to either accelerate or intensify development by reallocating funds during the initial design/development phases of a system acquisition cycle are frequently lost. Consequently, managers can lose an opportunity to support their projects at a time of greatest need.

There exists an abundance of reliability growth models designed to assist managers in charting the reliability status of a system and its components during any stage of development and testing. Unfortunately, the benefits, accuracies, and comparisons of these models are not always understood by management staff who could profit from their use.

The primary purpose of this thesis was to evaluate three easily understood, relatively simple reliability growth models which would hopefully prove to be reasonably accurate over a broad spectrum of reliability growth situations. Conditioned upon this demonstrated versatility in a wide variety of circumstances, the ultimate goal is

for employment and utilization of these reliability growth models by management on a significantly larger scale with increased willingness to share reliability progress information between contractor and customer. The evaluation was intentionally not structured to ascertain the capabilities of the reliability growth models as high resolution tools in very restricted situations but rather as gross models for flagging significant deviations of a system's or component's reliability progress from a desired or expected reliability growth pattern. Hence, the models' ability to alert managers that consideration of corrective action or at least more intense scrutiny of reliability status is required was established as a goal of the evaluation. Perhaps the greatest potential windfall benefit of utilizing capable reliability growth models as a project management tool is a reduction in cost overruns that have plagued weapons systems development since the 1950's. A primary cause of weapons systems cost overruns as noted in reference 3 is trying to develop and produce the systems too fast.

Since the underlying reliability progress pattern of a new system and its components is unknown to the project officer and contractor, these managers essentially need a "model for all seasons". In this vein the attributes desired in a reliability growth model are accuracy and robustness. Hence, a model(s) that can discern the true underlying reliability progress pattern with some degree of fidelity (accuracy) without sacrificing the capacity to cope with a significantly diverse range of reliability progress patterns (robustness) is the tool managers seek.

With these attributes in mind a continuous cumulative failure rate reliability growth model, a continuous instantaneous failure rate reliability growth model, and a discrete reliability growth model were chosen

for evaluation. Since each model is structured to measure a different parametric indicator of reliability progress, no direct comparisons between the models' performance were made. The models are presumptuous as the generally accepted title reliability growth models implies. In order to evaluate the models' accuracy and robustness not only were they tested against "nice" underlying reliability progress paths, but also, they were confronted with underlying patterns which exhibited reliability stagnation and degradation prior to continued growth as well as patterns demonstrating no reliability progress.

The three reliability growth models evaluated in this thesis are designed to be very general in the nature of the items for which they propose to chart reliability progress. The models can be employed for reliability tracking at the system level, sub-system level, and/or the individual component level during any phase of an acquisition cycle to include even retrofit programs. Hence, for the balance of this thesis "component" and "item" are used to reference an entire system, or a sub-system, or an individual component for which reliability progress is being modelled as appropriate.

The evaluation technique utilized computer simulation which permits the underlying reliability progress patterns to be specified while observing the models' performance in both determining and forecasting the underlying progress patterns. The simulation technique and evaluation measures are described in the next chapter.

II. EVALUATION METHOD AND MEASURES OF EFFECTIVENESS

A. EVALUATION METHOD

Performance evaluation of both continuous failure rate reliability growth models and the discrete reliability growth model was accomplished using computer Monte Carlo simulation. Computer simulation permits the analyst to specify and control the underlying reliability path of growth. By utilizing simulation data that is equivalent to data normally available in a testing and development program the models' performance may be compared with the specified underlying reliability path.

The continuous cumulative failure rate reliability growth model and the continuous instantaneous failure rate reliability growth model both utilize the same data from a testing and development program. Therefore, these two models were evaluated simultaneously in a unified simulation program. The test program assumed for the continuous models is essentially a deterministic program in which testing of items in any given test phase can be accomplished simultaneously since the number of items to be tested within each phase is specified prior to conduct of the test phase. On the other hand the discrete reliability growth model is based on a sequential test structure in which the number of units tested cannot be specified beforehand; specifically, testing in each phase is continued until a specified number of failures are observed which in turn generate a significant development or production change that hopefully improves reliability. Consequently, the number of tests within a test phase are random. Because of this dissimilarity in test program

structure for the continuous and discrete models, a separate simulation program was run for evaluation of the discrete reliability growth model.

B. MEASURES OF EFFECTIVENESS

As indicated in the introduction, the primary goal of this evaluation was to demonstrate the capability of selected reliability growth models to estimate unknown, underlying reliability progress paths with sufficient accuracy and versatility. Thus, the capability of the models to satisfy these requirements is a question of their accuracy and precision; i.e., (1) how close do the models' reliability or failure rate estimates approximate the true underlying reliability or failure rate paths and (2) are the models consistent in their approximations?

To answer these two questions regarding the models' capabilities the simulations were replicated one-hundred times for each underlying reliability progress path specified. The number of replications was actually programmed as a "user specified" input variable and could have been chosen to be ten runs for one progress path and a thousand runs for another progress path. One-hundred replications for all progress paths was chosen as a sufficiently large number to provide confidence in the test results while holding computer processing time requirements within acceptable limits. Two simple statistics were then computed for each model's performance on each specified underlying reliability progress path.

First, the average values (arithmetic means) of the models' estimates of the parameter utilized in the characterization of the underlying reliability progress path were computed to provide a measure of the models' accuracy. Average values of reliability or failure rate estimates were

computed for both the current reliability status at the end of a test phase and the forecast reliability status at the end of the following test phase. While a summary of these parameter mean value results could be presented in tabular form, such mode of presentation is difficult and time consuming to comprehend. Therefore, a graphical presentation was selected over the tabular form. The graphical presentation permits easy qualitative evaluation of a model's accuracy and recognition of estimate trends such as consistent optimistic/pessimistic estimation of the reliability progress path.

For each particular reliability growth model, test structure, and specified underlying reliability progress path, a graph was prepared depicting the true underlying path, the model's mean estimated value for the reliability or failure rate at the end of each test phase, and the model's mean forecast reliability or failure rate for the end of the following test phase. Since a total of 198 different combinations were simulated, graphs for all the simulation results are not presented. In those cases where no significant change in accuracy or trend results occurs graphs are omitted.

Project managers are concerned with only one "replication" of the reliability progress path estimation task as opposed to ten, one-hundred, or a thousand progress path estimates. Given that the mean performance of a reliability growth model is satisfactory, managers need to know if the model can be trusted to deliver satisfactory performance on that single "replication". This concern equates to the question of variability (precision) in the model estimates; i.e., does the model deliver "tight groups" around the mean values of its estimates? To measure variability the standard deviation of each model's reliability or failure

rate estimates from the mean value performance was computed for all phases of all simulations.

Since standard deviation is a relative measure, it is difficult to recognize trend in the performance of the reliability growth models from a table of standard deviation statistics. So, variability performance of the reliability growth models is presented in tabular form containing entries of standard deviation of the model estimates expressed as percentages of the magnitude of the reliability or failure rate values to which they correspond. (This statistic is the coefficient of variation expressed as a percentage; i.e., the percentage standard error.) Because gross performance characteristics of the reliability growth models were being evaluated, all percentages were truncated to the whole percentage point to facilitate examination of the variability performance tables for trends.

In examining the variability performance of the reliability growth models some guideline is desirable. In the field of econometric models a percentage standard error of 10-15% is the goal given by reference 6. In cost estimation modelling a percentage standard error of 15-20% is considered satisfactory according to reference 8. A reliability growth model that exhibits good mean estimation performance but displays very poor variability performance may be useless for practical purposes. On the other hand a reliability growth model that produces excellent variability performance coupled with poor but consistently biased mean estimation performance may prove to be very useful. Ultimately, the project manager will have to decide if the performance characteristics displayed by the reliability growth models examined in the next chapter qualify these models for application to his project.

III. RELIABILITY GROWTH MODEL DESCRIPTIONS AND PERFORMANCE RESULTS

A. CONTINUOUS CUMULATIVE FAILURE RATE RELIABILITY GROWTH MODEL

1. Model Description

The continuous cumulative failure rate model evaluated is of the form

$$\lambda_{TT} = \frac{\beta}{TT^{\alpha}} \tag{3.1}$$

where

λ_{TT} = cumulative failure rate of the item thru time TT,

TT = total accumulated test time for all testing, and

α, β = constants that determine the shape of the model estimated reliability progress path.

Cumulative failure rate may also be thought of as average failure rate or characteristic failure rate. Under the assumption that α is positive, the model relationship is that as test time is accumulated on an item (and by implication discrepancies discovered in the item are corrected) the denominator of the right-hand side of equation 3.1 increases; and hence, the cumulative failure rate of the item decreases. So reliability growth is modelled as decreasing failure rate by this model. Total accumulated test time TT is measured in whatever units are appropriate to the item being tested; i.e., minutes, hours, days, years, etc. or mission units. The model is considered continuous since the single independent variable TT , total accumulated test time, is continuous

as opposed to a discrete variable such as the number of failures experienced or the number of item modifications accomplished.

If the failure rate being experienced during any particular phase of testing is thought of as the instantaneous failure rate, then within the context of reliability growth instantaneous failure rate is expected to decrease as testing proceeds from phase to phase just as cumulative failure rate is expected to decrease. Since cumulative failure rate is a characterization of the failure rate of the item based on all the testing that has been accomplished; i.e., an "average" measure of failure rate, then cumulative failure rate is expected to be greater than the instantaneous failure rate at any point after the initial test phase when the two measures are equal. Conversely, if the reliability of the item decreases as the acquisition cycle develops (increasing instantaneous and cumulative failure rate), then cumulative failure rate is expected to be less than the instantaneous failure rate being experienced at any phase after the initial phase of testing.

A mathematical analysis of the cumulative failure rate model is given in reference 2.

2. Reliability Testing Procedure

For the continuous failure rate reliability growth models the appropriate reliability testing procedure of a system acquisition model is one that is conducted in distinct test phases. These test phases are indexed by i with a total of K phases comprising the duration of the procedure; i.e., i = 1, 2, 3, ... , K. Within each phase of a testing a specified number of tests are conducted where the number of tests is denoted by NT_i. The number of tests per phase NT_i can vary from phase to phase. During each phase of testing an underlying, inherent

failure rate that is unknown to the project managers/contractors is present in the item under test. This underlying failure rate is the instantaneous failure rate of the item for each phase and is denoted as λ_i . Also, during each phase of testing a phase total test time is accumulated and labeled as T_i . Hence, at the completion of test phase i the total accumulated test time TT_i on the item is given by

$$TT_i = \sum_{j=1}^{i} T_j \qquad (3.2)$$

for $i = 1, 2, 3, \ldots, K$ and $TT_1 = T_1$.

For each phase of the testing program a planned test time is established beforehand after which items under test that are still functioning are judged successful; i.e., non-failures. The planned test time for test phase i is denoted as PTT_i . Therefore, the maximum test time that can be accumulated during any given test phase i is given by the product $NT_i \times PTT_i$ and occurs only when none of the NT_i components tested fail. During each phase of the testing the number of failures that does occur is recorded as F_i . At the end of test phase i the total number of failures experienced for the entire testing procedure thru phase i is given by

$$TF_i = \sum_{j=1}^{i} F_j \qquad (3.3)$$

for $i = 1, 2, 3, \ldots, K$ and $TF_1 = F_1$.

Figure 3.1 is a schematic diagram of the continuous models reliability testing procedure. The total number of test phases K, the number of tests conducted during each test phase NT_i, and the planned test time

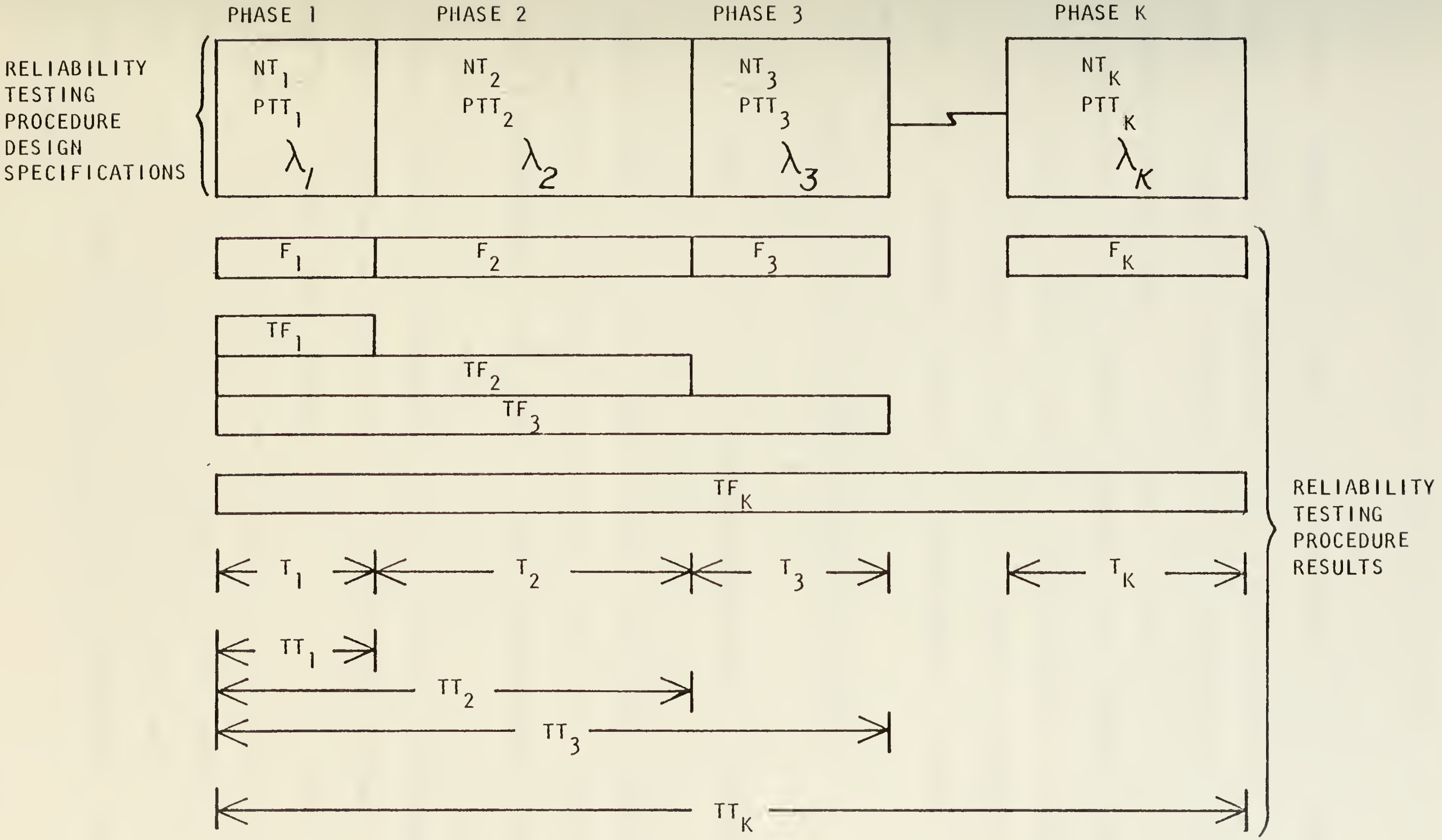

FIGURE 3.1

CONTINUOUS FAILURE RATE RELIABILITY GROWTH MODELS RELIABILITY TESTING PROCEDURE DIAGRAM

PTT_i at which an item under test is considered successful are determined prior to testing. The test time T_i accumulated during each test phase, the total accumulated test time TT_i at the end of each test phase, the number of failures F_i experienced during each test phase, and the total number of failures TF_i accumulate for the entire testing procedure thru the end of each test phase are all data collected from the reliability testing procedure. The most salient feature of the computer simulation of this reliability testing procedure is that the analyst also specifies the underlying instantaneous failure rate λ_i that is effective for each phase. Thus an accurate assessment of the accuracy of the model can be made because the λ_i values are known.

3. Reliability Testing Procedure Computer Simulation

a. Summary

The reliability test procedure appropriate to the continuous failure rate reliability growth models was simulated on the Naval Postgraduate School W. R. Church Computer Center's IBM 360/67 System utilizing standard computer simulation techniques. Reliability testing procedure design parameters K, NT_i, and PTT_i were specified along with various underlying instantaneous failure rate sets. Tests of components were then simulated with component success or failure being determined for each test based on the appropriate underlying instantaneous failure rate λ_i and the planned test time PTT_i . T_i, TT_i, F_i, and TF_i test data were collected as each testing procedure simulation was accomplished. Simulations of reliability testing procedures for each specified instantaneous failure rate set were replicated one-hundred times in order to assess the continuous failure rate reliability growth models' variability performance.

After the reliability testing procedure simulations were completed, continuous failure rate reliability growth model estimates of the underlying cumulative and instantaneous failure rate of the component being tested were produced based on the simulation test data by computations utilizing ordinary least squares regression techniques. Simple mean and variability statistics of these model estimates were then computed to permit performance evaluation of the continuous failure rate models.

b. Detail Description

The computer simulation of the continuous cumulative and the continuous instantaneous failure rate reliability growth models was initiated by reading in the following reliability testing procedure design specifications: K, the total number of test phases; NT_i, the number of tests to be conducted during each test phase; λ_i, the underlying instantaneous failure rate effective during each test phase; and NSIMS, the number of times the specified reliability testing procedure was to be simulated. For this thesis NSIMS was always specified as one-hundred simulations.

Rather than specify the test phase planned test times PTT_i at which components still operating were considered not to have failed for each phase, planned test times were calculated for each test phase such that

$$\exp(-\lambda_i \times PTT_i) = 0.7 \text{ or}$$

$$PTT_i = -\frac{1}{\lambda_i} \ln 0.7 . \qquad (3.4)$$

The lifetimes of components tested during each test phase i were taken to be exponentially distributed with mean time to failure (MTTF) $1/\lambda_i$.

The planned test times given by equation 3.4 result in each component tested having a 70% chance of surviving the test phase. This figure for survival probability of 0.7 was chosen arbitrarily to insure that some component failures were generated.

The procedure for specifying planned test times PTT_i admittedly deviates from actual test procedure design since the calculation in equation 3.4 is dependent upon the underlying instantaneous failure rate which is unknown in real life testing. However, if a reliability testing procedure is being conducted under the assumption of reliability growth, planned test times PTT_i would naturally be specified as increasing from test phase to test phase as the expected failure rate decreases. If the true underlying failure rate should stagnate or worsen from one test phase to the next, then the probability that components fail during the next test phase increases since planned test time will have been lengthened. Consequently, more failures would generally result than if the underlying reliability progress path had followed the expected "nice" growth pattern; i.e., decreasing failure rate.

After specification data were read into the computer, uniform (0,1) random variates were drawn; one corresponding to each component tested in each of the K total test phases; i.e., uniform random variates were drawn and indexed as $U_{i,j}$ for i = 1, 2, 3, ... , K and j = 1, 2, 3, ... , NT_i. For this evaluation the life length of components was assumed to be exponentially distributed. The assumption is a strong one, and the performance characteristics of the models under other than the exponential lifetime distribution assumption might prove to be a fruitful area for evaluation. Next, the uniform random variates were transformed into exponential lifetimes for the components under test via the probability

integral transform as given in reference 4; i.e., each component was assigned a test lifetime $E_{i,j}$ according to

$$E_{i,j} = -\frac{1}{\lambda_i} \ln U_{i,j} \tag{3.5}$$

for i = 1, 2, 3, ... , K and j = 1, 2, 3, ... , NT_i.

The number of failures F_i that occurred during each test phase was then tallied by comparing the test lifetime $E_{i,j}$ of each component with the appropriate planned test time PTT_i such that

$$F_i = \sum_{j=1}^{NT_i} \delta_j \quad \text{where} \quad \delta_j = \begin{cases} 1 \text{ if } E_{i,j} < PTT_i \text{ (Failure)} \\ \\ 0 \text{ if } E_{i,j} \geqq PTT_i \text{ (Success)} \end{cases} \tag{3.6}$$

for i = 1, 2, 3, ... , K. Similarly, the test time T_i accumulated for each test phase was recorded as

$$T_i = \sum_{j=1}^{NT_i} \varepsilon_{i,j} \text{ where } \varepsilon_{i,j} = \begin{cases} E_{i,j} \text{ if } E_{i,j} < PTT_i \\ \\ PTT_i \text{ if } E_{i,j} \geqq PTT_i \end{cases} \tag{3.7}$$

for i = 1, 2, 3, ... , K. Finally, the corresponding total failures TF_i and total test times TT_i accumulated thru the end of each test phase were tallied as

$$TF_i = \sum_{j=1}^{i} F_j \text{ , and} \tag{3.8}$$

$$TT_i = \sum_{j=1}^{i} T_j \tag{3.9}$$

for i = 1, 2, 3, ... , K respectively.

In keeping with accepted reliability growth modelling practice for reliability testing procedures utilizing planned test times, in those cases where no failures were experienced during a given test phase ($F_i = 0$) the number of failures for that test was assigned as one-half a failure; i.e., $F_i = \frac{1}{2}$. Similarly, whenever the total failures accumulated thru a given test phase were zero ($TF_i = 0$) based on the original failures per test phase statistics ($TF_i = \sum_{j=1}^{i} F_j = 0$), then total failures accumulated thru that phase were assigned as one-half a failure; i.e., $TF_i = \frac{1}{2}$. The primary thrust of these modifications is that zero failure statistics result in failure rate estimates of 0.0 which are extremely unrealistic and would virtually never occur under the exponential lifetime distribution assumption if infinite planned test times were possible. Hence, the one-half failure assignment procedure results in optimistic but positive failure rate estimates for those instances in which no failures are experienced.

These data constituted the total data gathered from a single computer simulation of the continuous model reliability testing procedure for a single specified underlying reliability progress path; i.e., a specified set of failure rates which shall henceforth be referred to as a lambda set. This computer simulation was repeated one-hundred times (NSIMS = 100) for each specified lambda set where the simulation replication is indexed as r = 1, 2, 3, ... , NSIMS = 100. Therefore, all the simulation data are indexed by the simulation number r which has been left implicit for simplification; i.e., the simulation results are $F_{i,r}$, $TF_{i,r}$, $T_{i,r}$, and $TT_{i,r}$. Since twenty-eight different lambda sets were utilized for evaluating the continuous failure rate reliability growth models with three different number of tests per test phase NT_i specified

for each lambda set, 8400 reliability testing procedures were simulated for the continuous reliability growth models evaluation.

4. Computer Simulation Data Manipulation

The continuous cumulative failure rate reliability growth model is proposed as a model of the unknown, underlying cumulative failure rate λ_{TT} of a component as it proceeds thru a system acquisition cycle. In each reliability testing procedure computer simulation the true cumulative failure rate λ_{TT} that was in effect at the end of a test phase was determined by the test time weighted average of the underlying true instantaneous failure rates λ_i that were in effect thru the end of the given test phase. Thus, the true cumulative failure rate was determined as

$$\lambda_{TT_i} = \sum_{j=1}^{i} \lambda_j \frac{T_j}{TT_i} \qquad (3.10)$$

for i = 1, 2, 3, ... , K. It is against this true cumulative failure rate that the model's performance was measured.

At the end of a phase of testing an estimator of the instantaneous failure rate for that phase of testing is given by

$$\widehat{\lambda}_i = \frac{\text{Number of Failures Experienced During Phase } i}{\text{Test Time Accumulated During Phase } i} = \frac{F_i}{T_i} \qquad (3.11)$$

for i = 1, 2, 3, ... , K. In turn an estimator of the cumulative failure rate thru the end of a phase of testing is given by the test time weighted average of the instantaneous failure rate estimates; i.e.,

$$\widehat{\lambda}_{TT_i} = \sum_{j=1}^{i} \widehat{\lambda}_j \frac{T_j}{TT_i} \qquad (3.12)$$

for i = 1, 2, 3, ... , K. Note that since $TT_1 = T_1$, then $\widehat{\lambda}_{TT_1} = \widehat{\lambda}_1$.

The estimator of the cumulative failure rate thru the end of any test phase may also be stated as

$$\widehat{\lambda}_{TT_i} = \frac{\sum_{j=1}^{i} F_j}{\sum_{j=1}^{i} T_j} = \frac{TF_i}{TT_i} \tag{3.13}$$

for i = 1, 2, 3, ... , K by the following line of reason: $F_j = \widehat{\lambda}_j T_j$ by equation 3.11; and therefore,

$$TF_i = \sum_{j=1}^{i} F_j = \sum_{j=1}^{i} \widehat{\lambda}_j T_j \quad \text{by equation 3.8.}$$

So,

$$\widehat{\lambda}_{TT_i} = \frac{TF_i}{TT_i} = \frac{\sum_{j=1}^{i} \widehat{\lambda}_j T_j}{TT_i} = \sum_{j=i}^{i} \widehat{\lambda}_j \frac{T_j}{TT_i} \tag{3.14}$$

for i = 1, 2, 3, ... , K which is equivalent to equation 3.12. Since $TF_1 = F_1$ and $TT_1 = T_1$, then

$$\widehat{\lambda}_{TT_1} = \frac{TF_1}{TT_1} = \frac{F_1}{T_1} = \widehat{\lambda}_1 \frac{T_1}{TT_1} = \widehat{\lambda}_1 \frac{T_1}{T_1} = \widehat{\lambda}_1 . \tag{3.15}$$

The estimates of the instantaneous failure rate $\widehat{\lambda}_i$ and the cumulative failure rate $\widehat{\lambda}_{TT_i}$ are based on data from a reliability testing procedure utilizing planned test times PTT_i . Therefore, the estimates obtained from equations 3.11 and 3.13 are biased estimates, the bias being in the pessimistic direction. In particular all of the

instantaneous failure rate estimates and the cumulative failure rate estimates for the early phases of the testing procedure are biased too large in magnitude. To compensate for this estimator bias a bias correction factor derived in reference 7 was applied to the instantaneous failure rate estimates $\widehat{\lambda_i}$ and the cumulative failure rate estimates $\widehat{\lambda_{TT_i}}$ such that

$$\widehat{\lambda_i} = \frac{F_i}{T_i}\left(\frac{2NT_i}{1 + 2NT_i}\right) \quad \text{and} \tag{3.16}$$

$$\widehat{\lambda_{TT_i}} = \frac{TF_i}{TT_i}\left(\frac{2 \sum_{j=1}^{i} NT_j}{1 + 2 \sum_{j=1}^{i} NT_j}\right) \tag{3.17}$$

for $i = 1, 2, 3, \ldots, K$.

Next, estimates of α and β in the cumulative failure rate model, equation 3.1, were obtained using ordinary least squares regression techniques. Because ordinary least squares regression requires at least two observation points, estimates for α and β could only be derived from the model starting at the conclusion of phase two; i.e., estimates were derived for phases $1 = 2, 3, 4, \ldots, K$.

If the cumulative failure rate model in equation 3.1 is written as

$$\lambda_{TT_i} = \beta_i \, TT_i^{-\alpha_i} \tag{3.18}$$

and the logarithmic transformation is applied to this equation, then

$$\ln \lambda_{TT_i} = \ln \beta_i - \alpha_i \ln TT_i \, . \tag{3.19}$$

Letting

$$Y_i = \ln \widehat{\lambda}_{TT_i} \quad ,$$

$$a_i = \ln \beta_i \quad ,$$

$$b_i = -\alpha_i \quad ,$$

$$X_i = \ln TT_i \quad ,$$

$$\bar{X} = \frac{1}{i} \sum_{j=1}^{i} X_j \text{ , and}$$

$$\bar{Y} = \frac{1}{i} \sum_{j=1}^{i} Y_j \quad ,$$

then equation 3.19 is of the form

$$Y_i = a_i + b_i X_i \quad . \tag{3.20}$$

Note that the estimated cumulative failure rates $\widehat{\lambda}_{TT_i}$ from equation 3.17 are utilized in performing the regressions. Applying ordinary least squares regression estimates as given in reference 6 the $\widehat{a}_i$ and $\widehat{b}_i$ estimates are

$$\widehat{b}_i = \frac{\sum_{j=1}^{i} (X_j - \bar{X}) Y_j}{\sum_{j=1}^{i} (X_j - \bar{X})^2} \quad \text{and} \tag{3.21}$$

$$\widehat{a}_i = \bar{Y} - \widehat{b}_i \bar{X} \tag{3.22}$$

for i = 2, 3, 4, ... , K. The $\widehat{\alpha}_i$ and $\widehat{\beta}_i$ estimates are then obtained by the inverse transformations $\widehat{\alpha}_i = -\widehat{b}_i$ and $\widehat{\beta}_i = \exp(\widehat{a}_i)$. Finally,

these $\widehat{\alpha}_i$ and $\widehat{\beta}_i$ estimates are utilized in the cumulative failure rate model, equation 3.18, to produce the model estimates of the unknown, underlying cumulative failure rate. These estimates are given by

$$\widehat{\lambda^*_{TT_i}} = \frac{\widehat{\beta}_i}{TT_i^{\widehat{\alpha}_i}} \tag{3.23}$$

for i = 2, 3, 4, ... , K. For the first test phase the cumulative failure rate estimate is simply taken as the estimate given by equation 3.17; i.e.,

$$\widehat{\lambda^*_{TT_1}} = \widehat{\lambda_{TT_1}} = \widehat{\lambda_1} = \frac{F_1}{T_1} \left(\frac{2\ NT_1}{1 + 2\ NT_1} \right) \tag{3.24}$$

Again, since the reliability testing procedure was simulated one-hundred times (NSIMS = 100) for each lambda set, one-hundred estimates of the cumulative failure rate were obtained for each test phase. The mean value and standard deviation of these estimates were computed as

$$\overline{\widehat{\lambda^*_{TT_i}}} = \frac{1}{NSIMS} \sum_{r=1}^{NSIMS} \widehat{\lambda^*_{TT_{i,r}}} \quad \text{and} \tag{3.25}$$

$$S.D._{\widehat{\lambda^*_{TT_i}}} = \sqrt{\frac{1}{NSIMS - 1} \sum_{r=1}^{NSIMS} \left(\widehat{\lambda^*_{TT_{i,r}}} - \overline{\widehat{\lambda^*_{TT_i}}} \right)^2} \tag{3.26}$$

for i = 1, 2, 3, ... , K. Finally, the measure of variability for the cumulative failure rate model was computed as

$$P.S.E._{\widehat{\lambda^*_{TT_i}}} = \frac{S.D._{\widehat{\lambda^*_{TT_i}}}}{\overline{\widehat{\lambda^*_{TT_i}}}} \times 100 \tag{3.27}$$

for i = 1, 2, 3, ... , K.

Because the phase total test time T_i and the total accumulated test time TT_i are random variables which changed from simulation to simulation, their mean values were computed as

$$\overline{T_i} = \frac{1}{NSIMS} \sum_{r=1}^{NSIMS} T_{i,r} \quad \text{and} \tag{3.28}$$

$$\overline{TT_i} = \frac{1}{NSIMS} \sum_{r=1}^{NSIMS} TT_{i,r} \tag{3.29}$$

respectively for i = 1, 2, 3, ..., K. Similarly, the mean value of the true underlying test time weighted average cumulative failure rate was computed as

$$\overline{\lambda_{TT_i}} = \frac{1}{NSIMS} \sum_{r=1}^{NSIMS} \lambda_{TT_{i,r}} \tag{3.30}$$

for i = 1, 2, 3, ... , K. Figure 3.2 is a graphical representation of the results obtained from the reliability testing procedure for the continuous cumulative failure rate reliability growth model.

Equations 3.25 ($\overline{\widehat{\lambda^*_{TT_i}}}$) and 3.7 (P.S.E.$\widehat{\lambda^*_{TT_i}}$) are measures of the cumulative failure rate reliability growth model's performance in determining what a component's reliability status is at the end of a test phase. To examine the cumulative model's forecasting capability the following procedure was followed. The total accumulated test time TT_i of the cumulative model, equation 3.18 , is a stochastic (random) variable. Hence, at the end of any test phase i the total accumulated test time for subsequent phases of testing (TT_{i+1}, TT_{i+2}, ...) is unknown but

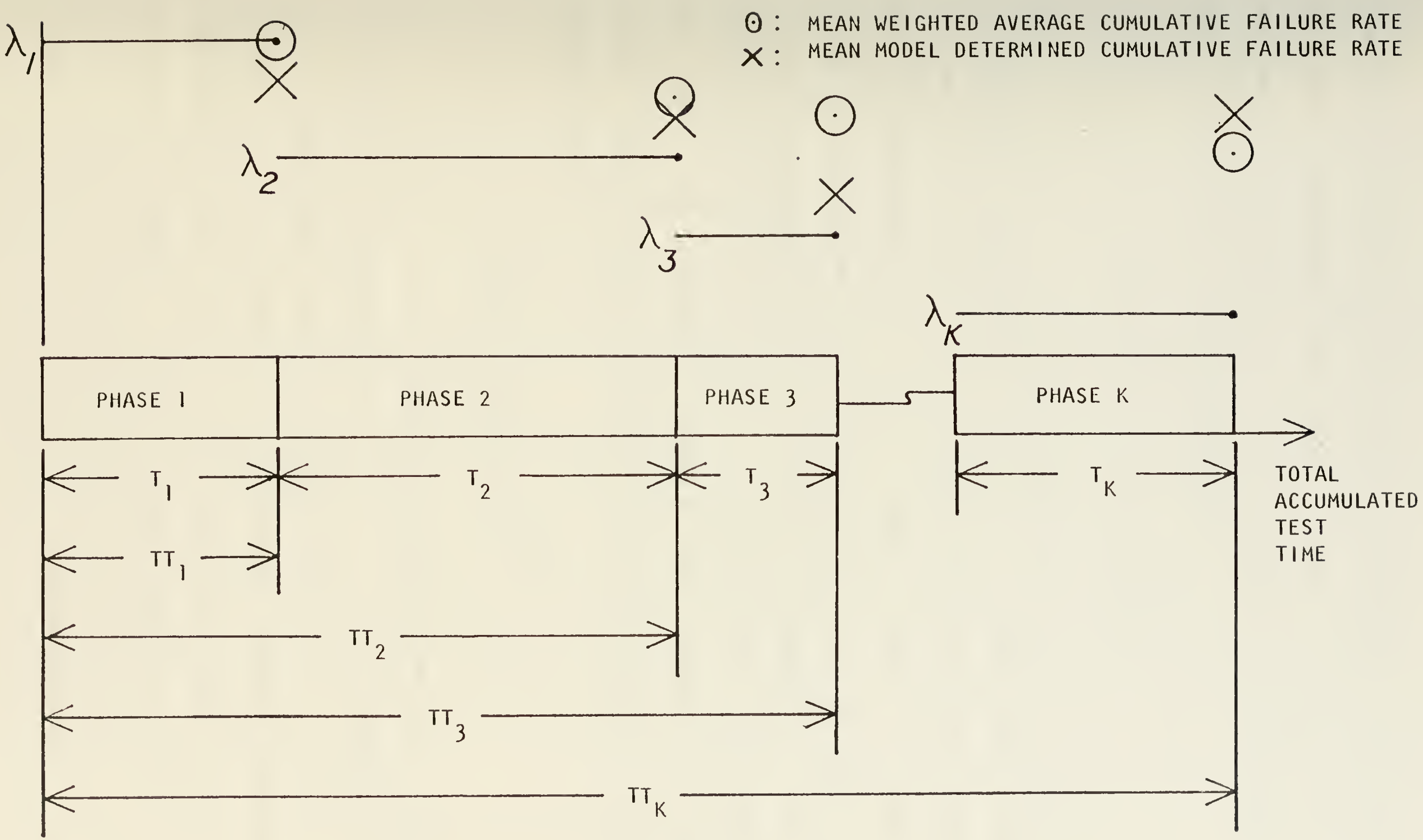
⊙ : MEAN WEIGHTED AVERAGE CUMULATIVE FAILURE RATE
× : MEAN MODEL DETERMINED CUMULATIVE FAILURE RATE
λ_1
λ_2
λ_3
λ_K
PHASE 1
PHASE 2
PHASE 3
PHASE K
T_1
T_2
T_3
T_K
TT_1
TT_2
TT_3
TT_K
TOTAL ACCUMULATED TEST TIME

FIGURE 3.2

GRAPHICAL REPRESENTATION OF RELIABILITY TESTING PROCEDURE RESULTS FOR CONTINUOUS CUMULATIVE FAILURE RATE RELIABILITY GROWTH MODEL

required if the model is to be used for forecasting. For this evaluation a one test phase in the future forecast of cumulative failure rate was made by approximating the next test phase (i+1) total accumulated test time from the current test phase as

$$\widehat{TT}_{i+1} = TT_i + (NT_{i+1} \times PTT_{i+1}) \tag{3.31}$$

for i = 2, 3, 4, ... , K. Thus at the end of a given test phase i the recorded total accumulated test time TT_i for that phase was added to the maximum phase total test time that could be accumulated during the following test phase i+1; i.e., the number of items to be tested NT_{i+1} times the planned test time PTT_{i+1} for the next test phase. This procedure thus approximated the most optimistic outcome; i.e., no failures for the following test phase i+1. The approximated total accumulated test time TT_{i+1} for the next phase was then utilized in the current phase i cumulative model to obtain a next phase forecast cumulative failure rate as

$$\widehat{F\lambda}_{TT_{i+1}} = \widehat{\beta}_i (\widehat{TT}_{i+1})^{-\widehat{\alpha}_i} \tag{3.32}$$

for i = 2, 3, 4, ... , K. Therefore, forecast cumulative failure rates were made for test phases i = 3, 4, 5, ... , K. No forecast could be made for test phase 2 because the model parameters α_i and β_i were not estimated until test phase 2 was completed. No forecast was made past test phase K since there would be no underlying cumulative failure rate with which to compare. Finally, forecast cumulative failure rate statistics similar to the determined failure rate statistics in equations 3.25, 3.26, and 3.27 were computed for the simulation replications (NSIMS = 100) as

$$\overline{\widehat{F\lambda}}_{TT_i} = \frac{1}{NSIMS} \sum_{r=1}^{NSIMS} \widehat{F\lambda}_{TT_{i,r}} \qquad (3.33)$$

$$S.D._{\widehat{F\lambda}_{TT_i}} = \sqrt{\frac{1}{NSIMS - 1} \sum_{r=1}^{NSIMS} (\widehat{F\lambda}_{TT_{i,r}} - \overline{\widehat{F\lambda}}_{TT_i})^2} \text{ , and} \qquad (3.34)$$

$$P.S.E._{\widehat{F\lambda}_{TT_i}} = \frac{S.D._{\widehat{F\lambda}_{TT_i}}}{\overline{\widehat{F\lambda}}_{TT_i}} \times 100 \qquad (3.35)$$

for $i = 3, 4, 5, \ldots, K$.

As a starting point for the evaluation of the continuous failure rate models the lambda sets utilized by Jayachandran and Moore in reference 5 were adopted as specified underlying instantaneous failure rate λ_i sets for five reliability progress paths. These lambda sets are listed in table 3.1 where they are numbered as lambda sets 1 thru 5. Lambda sets 1 thru 5 represent "nice" reliability progress paths since, although they vary in rate and span of failure rate decay, they have smooth exponential like decay patterns (curves). A decaying failure rate path is equivalent to a reliability progress path that displays growth.

Note in table 3.1 that for lambda set 3 for example the specified underlying instantaneous failure rate starts at 0.700 for test phase 1, decreases to 0.3530 during test phase 2, and finally reaches a value of 0.0500 for the last test phase, test phase 16.

Because a primary goal of this thesis was to evaluate the robustness of the selected reliability growth models, the "nice" lambda sets 1 thru 5 were augmented with nine anomalous lambda sets numbered 6 thru 14 which are also listed in table 3.1. Lambda sets 6 and 7 represent

TABLE 3.1

LAMBDA SETS

CONTINUOUS RELIABILITY GROWTH MODELS

	LAMBDA SET **********													
PHASE *****	1 ****	2 ****	3 ****	4 ****	5 ****	6 ****	7 ****	8 ****	9 ****	10 ****	11 ****	12 ****	13 ****	14 ****
1	.7020	.7000	.7000	.7000	.7000	.7000	.7000	.7000	.7000	.7000	.7000	.7000	.7000	.7000
2	.4340	.5730	.3530	.5640	.4560	.6734	.6800	.7000	.3000	.0500	.5250	.5500	.5500	.4500
3	.3200	.5080	.2380	.4600	.3150	.6468	.6600	.7000	.3000	.0500	.5100	.4250	.4000	.3000
4	.2550	.4660	.1800	.3790	.2270	.6202	.6400	.7000	.3000	.0500	.5000	.4050	.3500	.2250
5	.2130	.4350	.1450	.3150	.1700	.5936	.6200	.7000	.3000	.0500	.5000	.4000	.3100	.2000
6	.1830	.4110	.1220	.2630	.1320	.5670	.6000	.7000	.3000	.0500	.5000	.4000	.3000	.2250
7	.1610	.3920	.1060	.2210	.1060	.5404	.5800	.7000	.3000	.0500	.5000	.4000	.3100	.3000
8	.1440	.3760	.0933	.1860	.0880	.5138	.5400	.7000	.3000	.0500	.5000	.4000	.3500	.4000
9	.1300	.3620	.0837	.1570	.0757	.4872	.5000	.7000	.3000	.0500	.5000	.4000	.4000	.4750
10	.1180	.3500	.0760	.1320	.0672	.4606	.4500	.7000	.3000	.0500	.5000	.4000	.5500	.5500
11	.1090	.3400	.0697	.1120	.0613	.4340	.4000	.7000	.3000	.0500	.3250	.4000	.5900	.6100
12	.1010	.3300	.0644	.0953	.0572	.4074	.3500	.7000	.3000	.0500	.3100	.4000	.6000	.6250
13	.0936	.3220	.0600	.0810	.0544	.3808	.3000	.7000	.3000	.0500	.3000	.3750	.5900	.6100
14	.0876	.3140	.0562	.0689	.0524	.3542	.2500	.7000	.3000	.0500	.3000	.2000	.5000	.5500
15	.0823	.3070	.0529	.0587	.0510	.3276	.2000	.7000	.3000	.0500	.3000	.1000	.4000	.3000
16	.0776	.3010	.0500	.0500	.0500	.3000	.0500	.7000	.3000	.0500	.3000	.0500	.3000	.0500

approximately linear underlying reliability progress paths. Lambda sets 8, 9, and 10 portray underlying reliability progress paths in which reliability rapidly attains a certain level and stagnates at that level for the duration of the reliability testing procedure (acquisition cycle). Lambda sets 11 and 12 display initial "nice" reliability growth followed by a period of reliability stagnation and a subsequent continued growth pattern. Finally, lambda sets 13 and 14 represent underlying reliability progress paths in which reliability growth is interrupted by a period of reliability degradation.

As indicated in table 3.1 lambda sets 1 thru 14 each represents reliability testing procedures of systems acquisition cycles consisting of sixteen test phases (K = 16). For very expensive and complex weapons systems a sixteen test phase procedure may not be feasible. Therefore, lambda sets for a more realistic six test phase (K = 6) cycle were also utilized in the computer simulation evaluation. These lambda sets for a contracted reliability testing procedure are listed in table 3.2 and numbered MOD1 thru MOD14. Lambda sets MOD1 thru MOD14 represent underlying reliability progress patterns that are equivalent to the progress patterns characterized by lambda sets 1 thru 14 but which develop during a six test phase cycle rather than over a sixteen test phase cycle.

In order to examine the continuous failure rate reliability growth models for a representative set of acquisition programs, procedures with items tested per test phase NT_i of five, ten, and twenty items were simulated utilizing all twenty-eight lambda sets; i.e., $NT_i = 5, 10,$ and 20 for $i = 1, 2, 3, \ldots, 16$ or $i = 1, 2, 3, 4, 5, 6$ as appropriate. Hence, $28 \times 3 \times 100 = 8400$ reliability testing procedures were simulated for the continuous failure rate reliability growth models evaluation.

TABLE 3.2

MODIFIED LAMBDA SETS
CONTINUOUS RELIABILITY GROWTH MODELS

	LAMBDA SET **********													
PHASE *****	MOD1 ****	MOD2 ****	MOD3 ****	MOD4 ****	MOD5 ****	MOD6 ****	MOD7 ****	MOD8 ****	MOD9 ****	MOD10 *****	MOD11 *****	MOD12 *****	MOD13 *****	MOD14 *****
1	.7020	.7000	.7000	.7000	.7000	.7000	.7000	.7000	.7000	.7000	.7000	.7000	.7000	.7000
2	.2550	.4660	.1800	.3790	.2270	.6202	.6400	.7000	.3000	.0500	.5000	.4050	.3500	.2250
3	.1610	.3920	.1060	.2210	.1060	.5404	.5800	.7000	.3000	.0500	.5000	.4000	.3100	.3000
4	.1180	.3500	.0760	.1320	.0672	.4606	.4500	.7000	.3000	.0500	.5000	.4000	.5500	.5500
5	.0936	.3220	.0600	.0810	.0544	.3808	.3000	.7000	.3000	.0500	.3000	.3750	.5900	.6100
6	.0776	.3010	.0500	.0500	.0500	.3000	.0500	.7000	.3000	.0500	.3000	.0500	.3000	.0500

5. Model Performance

a. Accuracy Performance

Figures 3.3 thru 3.41 present selected cases of the continuous failure rate reliability growth model's capability to determine and forecast the unknown, underlying cumulative failure rate progress path during an acquisition cycle. These graphs depict the mean test time weighted average true underlying cumulative failure rate $\overline{\lambda}_{TT_i}$ progress path from equation 3.30 (——— , solid line), the mean model determined cumulative failure rate $\overline{\hat{\lambda}^*_{TT_i}}$ for each test phase from equation 3.25 (○, circles), and the mean model forecast cumulative failure rate $\overline{\widehat{F\lambda}}_{TT_i}$ for the third thru the final test phase from equation 3.33 (X , crosses) plotted versus the mean total accumulated test time $\overline{TT}_i$ from equation 3.29. Note that the point plotted for the test phase 1 mean model determined cumulative failure rate is not a mean model determined estimate; but rather, it is the mean estimate of the cumulative failure rate (and instantaneous failure rate since the estimate is for test phase 1) given by equation 3.24 ($\hat{\lambda}^*_{TT_1} = \hat{\lambda}_1$). So the accuracy of the reliability estimator of equation 3.24 may also be observed to a degree by examining the results for test phase 1.

To illustrate these quantities graphed in the accuracy performance figures note on figure 3.11 that at approximately 82.00 total accumulated test time units the mean weighted cumulative failure rate $\overline{\lambda}_{TT_{16}}$ was 0.30 while the mean model determined cumulative failure rate $\overline{\hat{\lambda}^*_{TT_{16}}}$ was approximately 0.45 and the mean model forecast cumulative failure rate $\overline{\widehat{F\lambda}}_{TT_{16}}$ was approximately 0.50. Since these values were the last points on the plot, they represent the simulation results for the sixteenth phase of a reliability testing procedure wherein five tests per

phase were conducted and for which lambda set 7 from table 3.1 specified the unknown, underlying reliability progress path. Also, note that the first mean model forecast cumulative failure rate point corresponds to the third mean model determined cumulative failure rate plotted; i.e., the first model forecast cumulative failure rate was for test phase 3 ($i = 3$). Finally, for test phase 1 the point plotted for the mean model determined cumulative failure rate is approximately 0.73 which is actually the mean value of the estimate produced by the instantaneous failure rate estimator of equation 3.24. Notice that the mean test time weighted average cumulative failure rate at test phase 1 is 0.70 which corresponds to the specified underlying instantaneous failure rate for this phase from table 3.1. This equivalence is explained by equation 3.15.

Figures 3.3 thru 3.9 and 3.25 thru 3.29 indicate that for "nice" underlying reliability progress paths the cumulative failure rate model displays very accurate performance with slight pessimistic bias (higher failure rate estimates). "Least data" graphs; i.e., five tests per phase cases ($NT_i = 5$) are generally displayed. Ten and twenty tests per phase ($NT_i = 10/20$) performance was as good as or better than five tests per phase for all cases in which only the $NT_i = 5$ graphs are shown. This consistency of performance is displayed for lambda set 3, $NT_i = 5/10/20$ sequence in figures 3.5, 3.6, and 3.7. Forecasting performance displays a general trend of overly pessimistic forecasts for the first two or three phases improving rapidly after four or five forecasts have been made.

Figures 3.10 thru 3.13, 3.30, and 3.31 reveal that the cumulative failure rate model has some difficulty in tracking linear underlying reliability growth patterns even when a relatively large amount

of data are available (figure 3.13, $NT_i = 20$). In all the linear cases both the model determined and model forecast mean failure rates diverge from the actual cumulative failure rate. Performance does not improve as the number of components tested per phase is increased as evidenced by the lambda set 7, $NT_i = 5/10/20$ sequence performance in figures 3.11, 3.12, and 3.13.

Figures 3.14 thru 3.20 and 3.32 thru 3.36 indicate that the cumulative failure rate model copes with permanently stagnated underlying reliability progress paths easily. The figure series 3.14 thru 3.16, 3.18 thru 3.20, and 3.32 thru 3.34 contrast the accuracy performance for increasing number of test per phase ($NT_i = 5/10/20$). Again, forecasting performance is initially very poor but quickly improves after three of four forecasts have been made.

In situations of the underlying reliability progress path experiencing temporary stagnation figures 3.21, 3.22, and 3.37 thru 3.39 reveal that the cumulative failure rate model, while picking up the stagnation credibly, has difficulty in accurately modelling the cumulative reliability improvement following the period of temporary stagnation. During this post stagnation improvement phase, the cumulative model's determined and forecast failure rate estimates diverge on the pessimistic side from the true underlying cumulative failure rate path much as they did in the linear underlying progress path cases. Figures 3.38 and 3.39 show that this trait is not relieved by gathering more test data per test phase of the reliability testing procedure ($NT_i = 5$ vs. $NT_i = 20$).

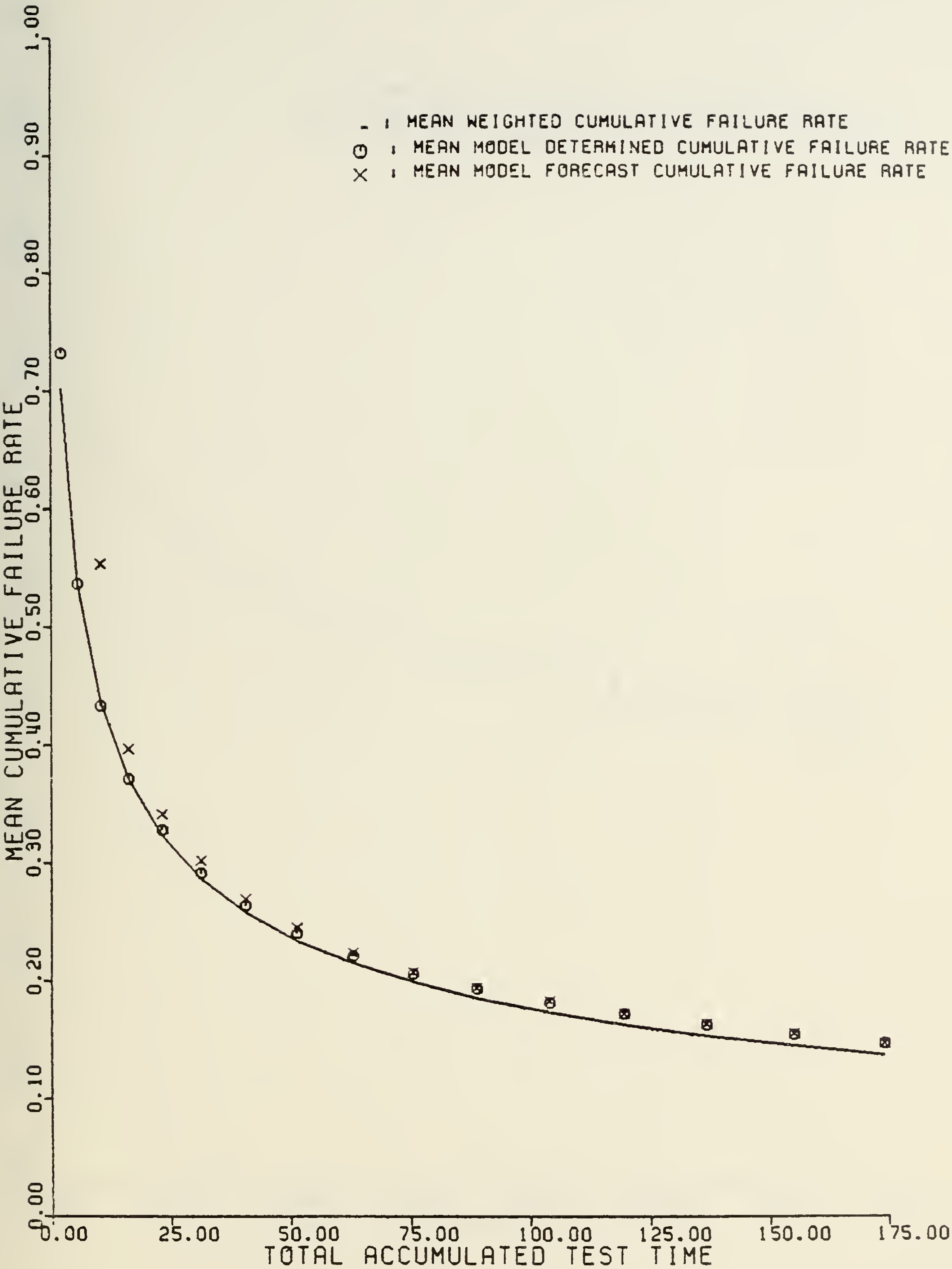

FIGURE 3.3

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 1: 16 PHASES, 5 TESTS/PHASE

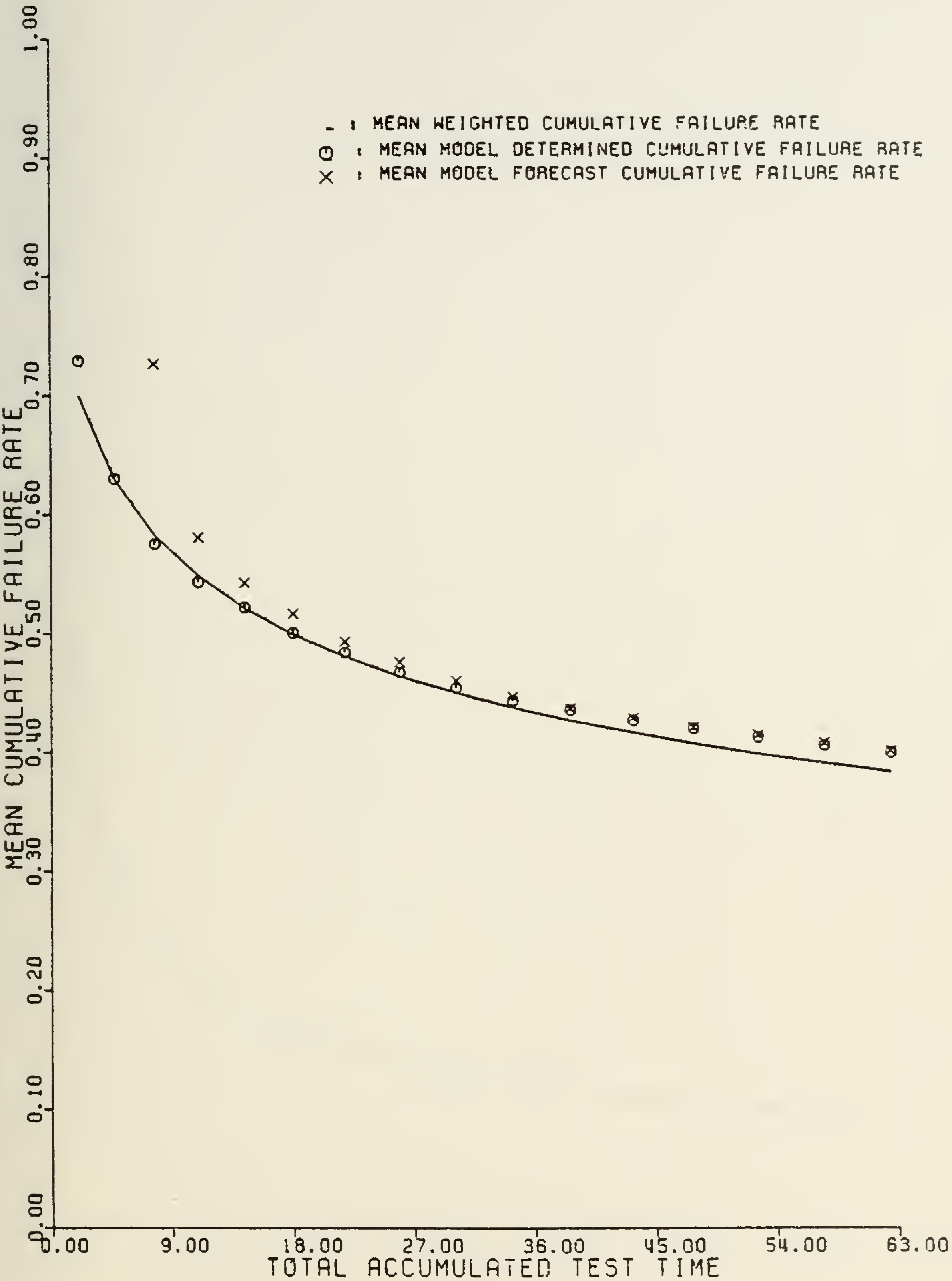

FIGURE 3.4

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 2: 16 PHASES, 5 TESTS/PHASE

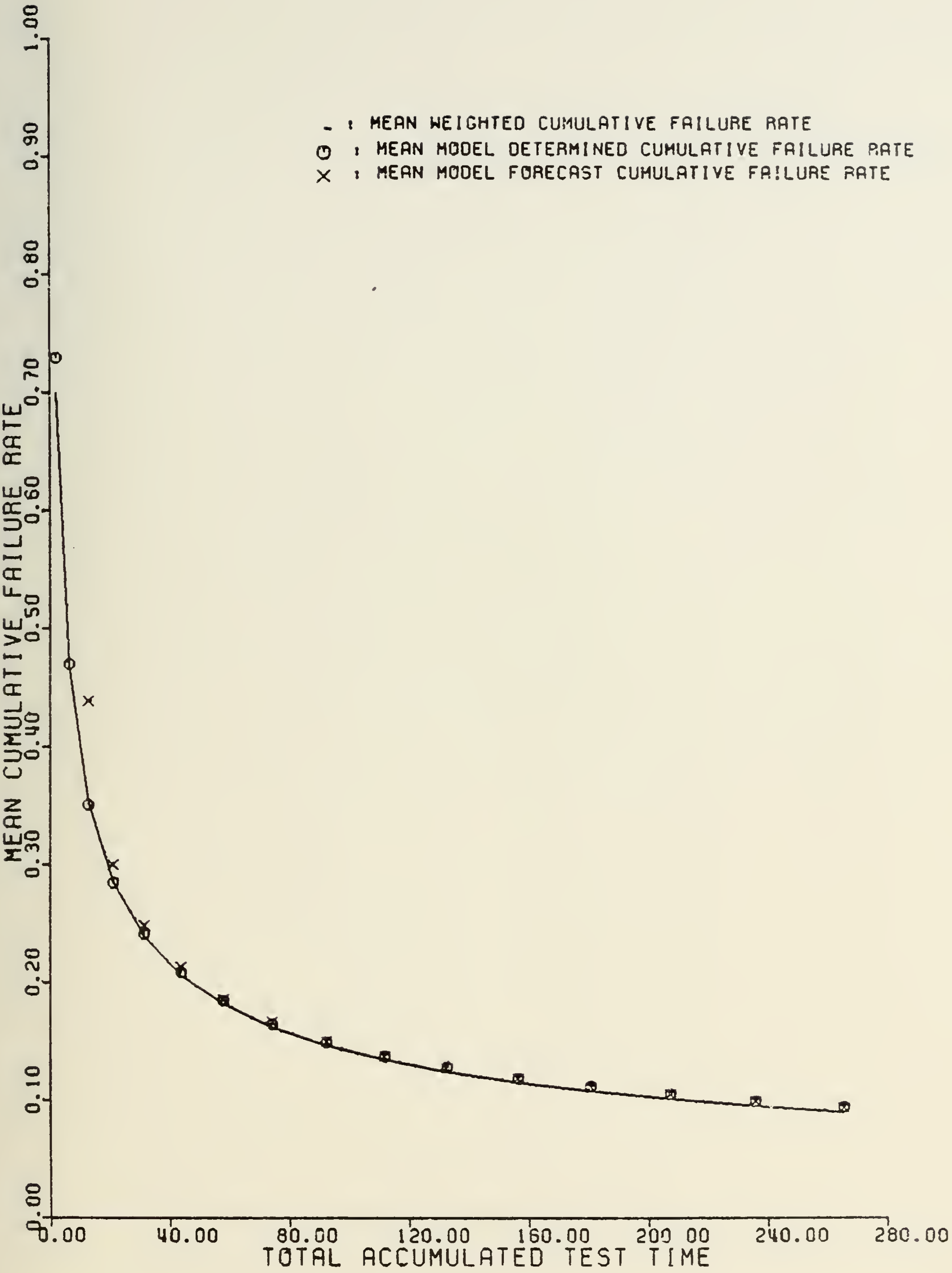

FIGURE 3.5

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 3: 16 PHASES, 5 TESTS/PHASE

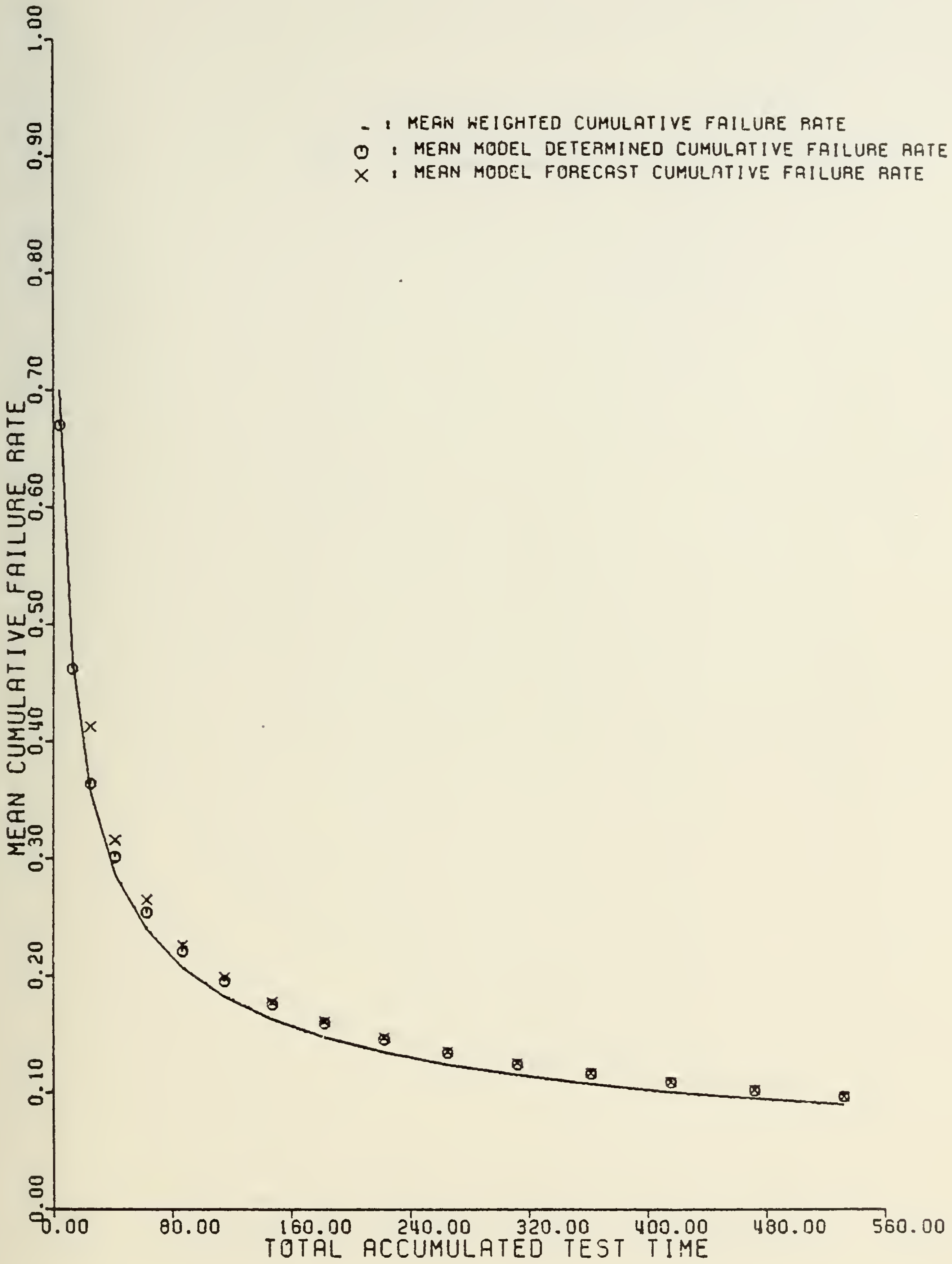

FIGURE 3.6

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 3: 16 PHASES, 10 TESTS/PHASE

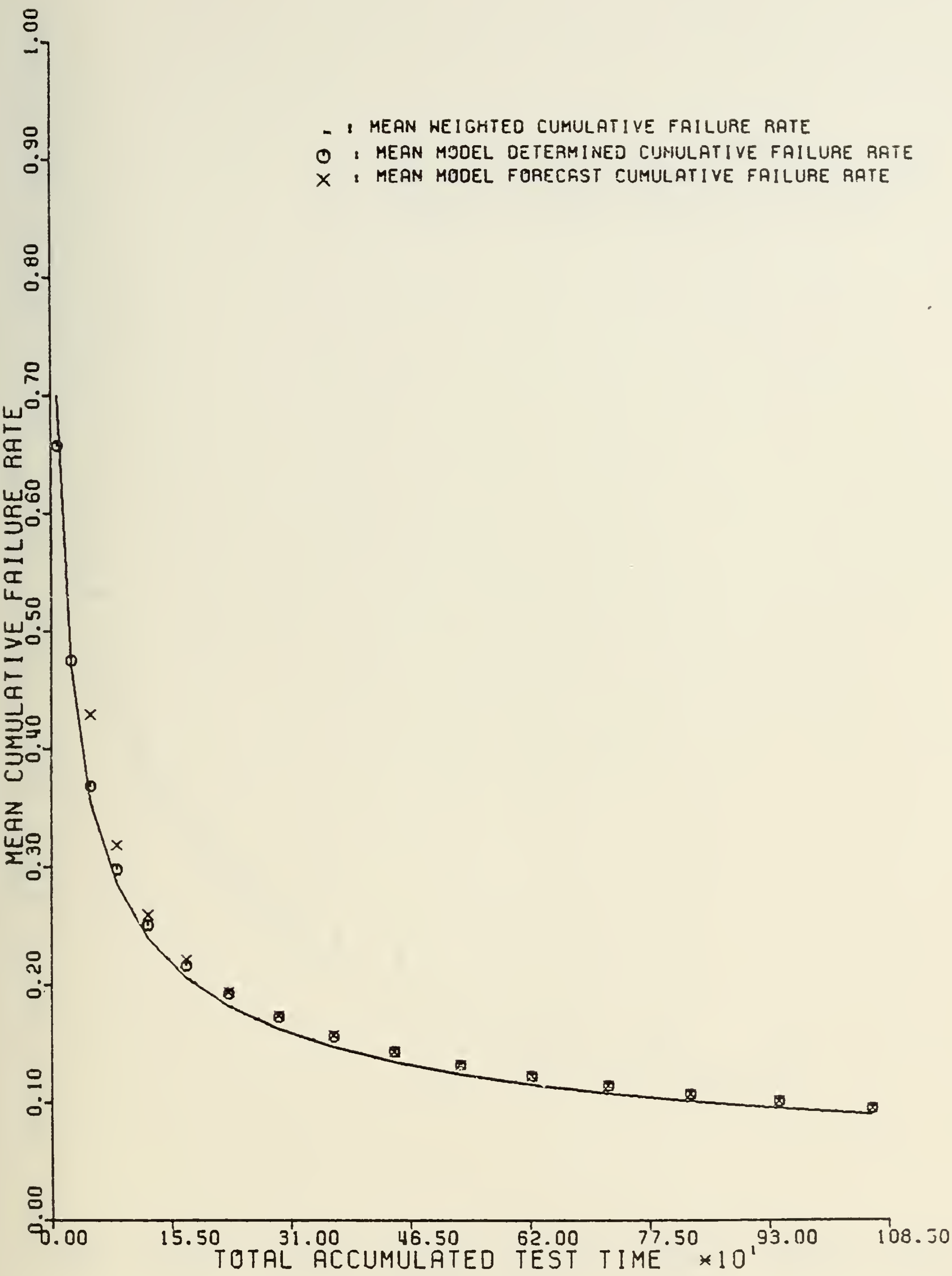

FIGURE 3.7

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 3: 16 PHASES, 20 TESTS/PHASE

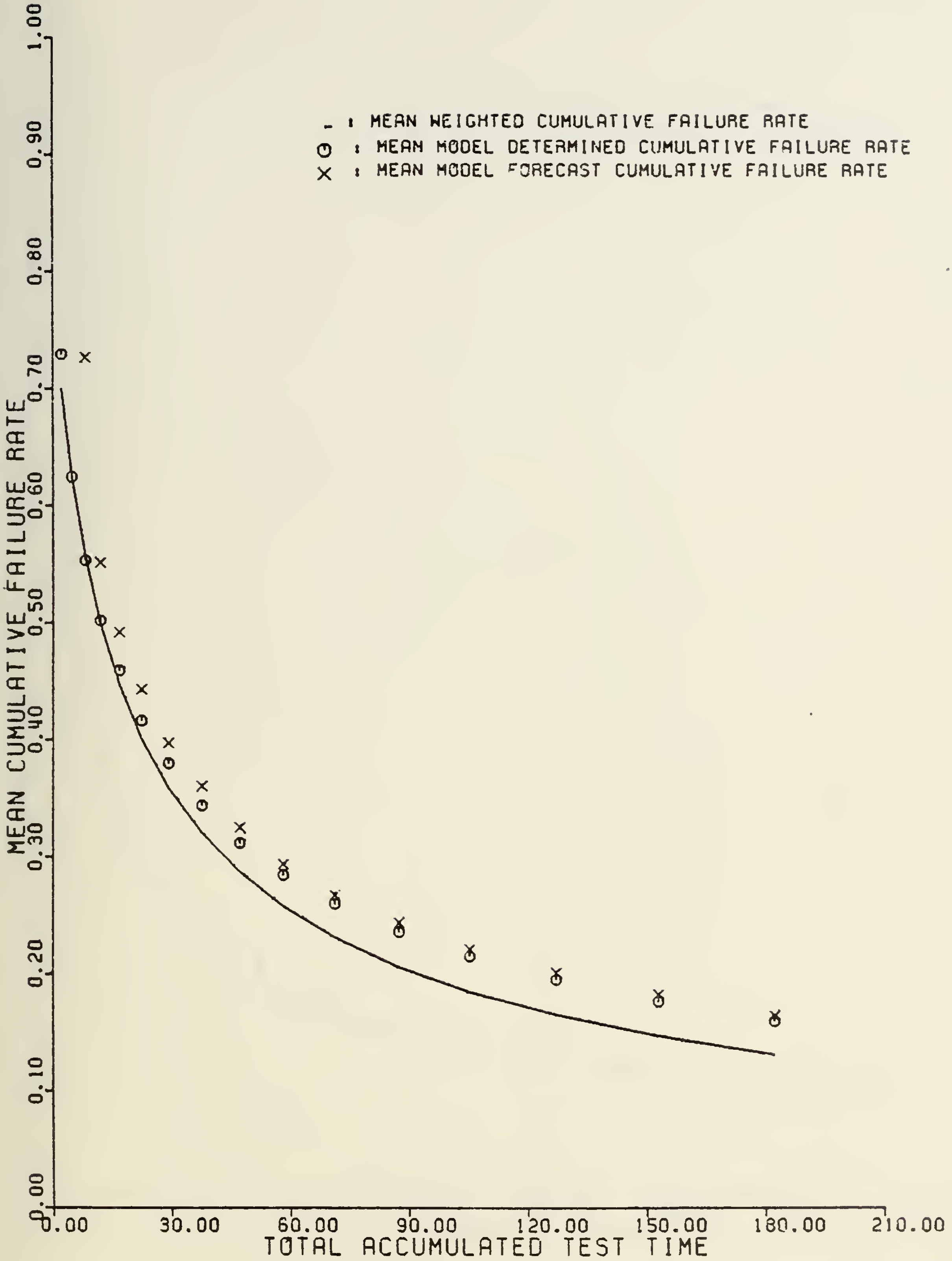

FIGURE 3.8

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 4: 16 PHASES, 5 TESTS/PHASE

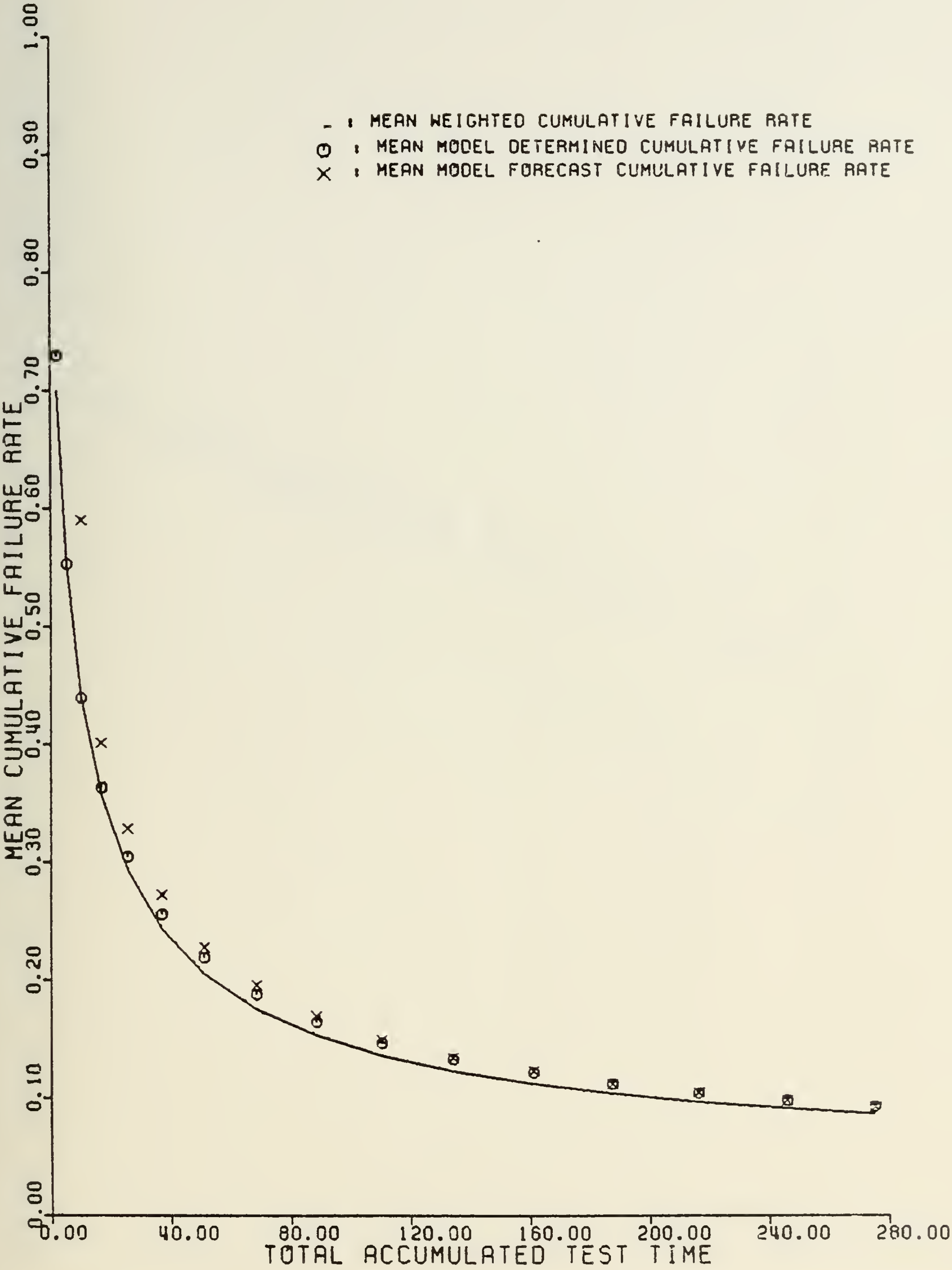

FIGURE 3.9

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 5: 16 PHASES, 5 TESTS/PHASE

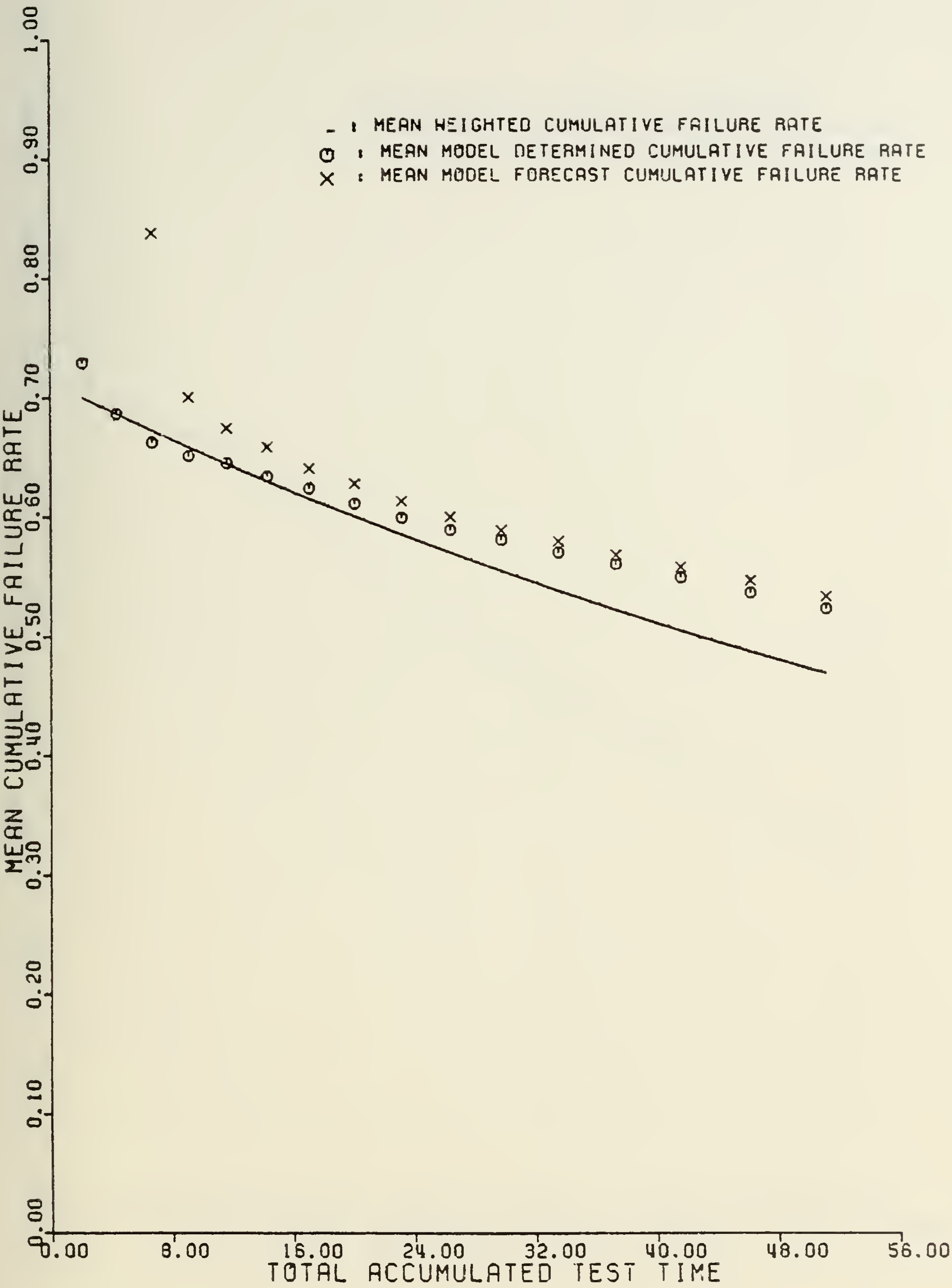

FIGURE 3.10

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 6: 16 PHASES, 5 TESTS/PHASE

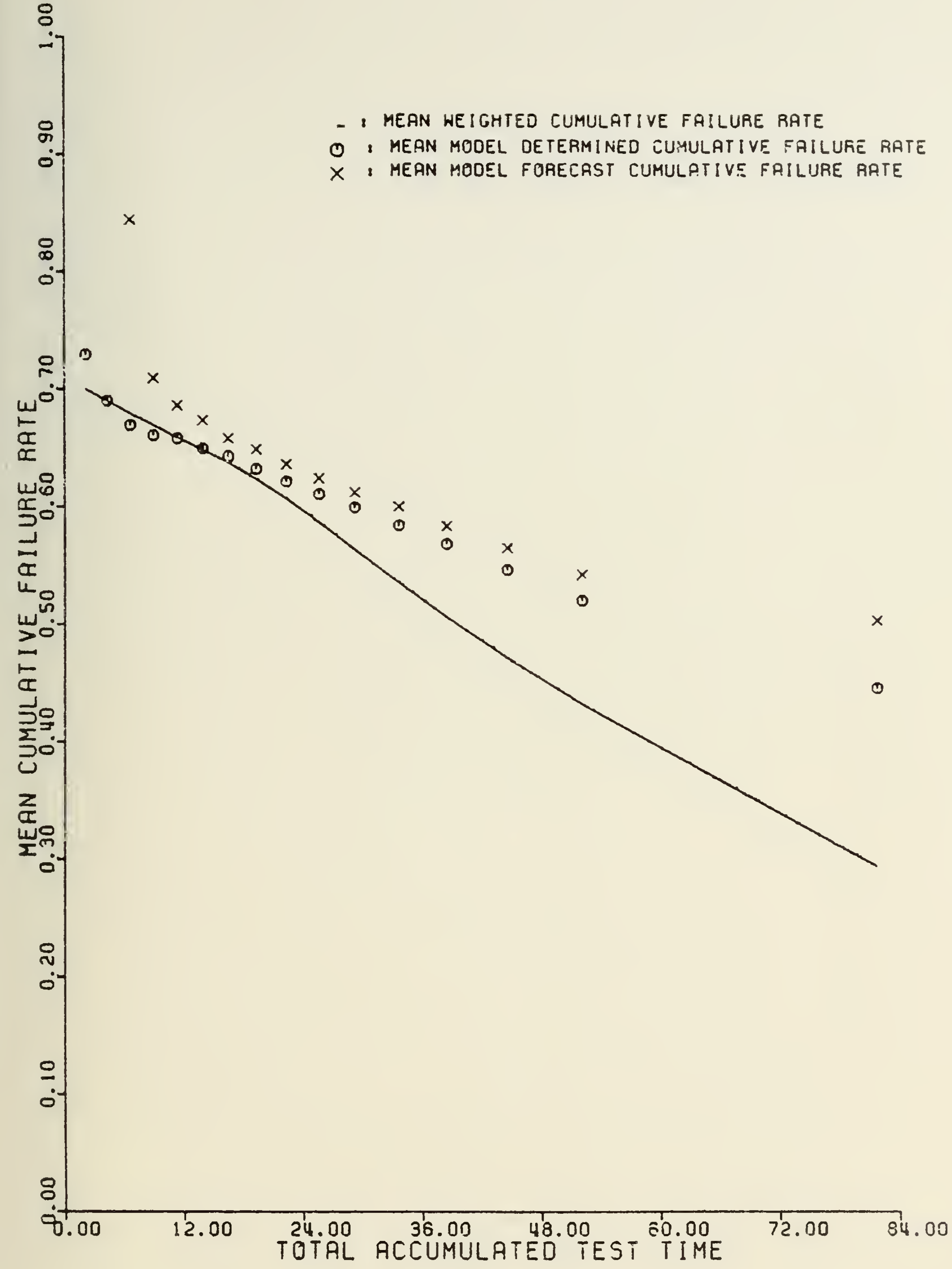

FIGURE 3.11
CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 7: 16 PHASES, 5 TESTS/PHASE

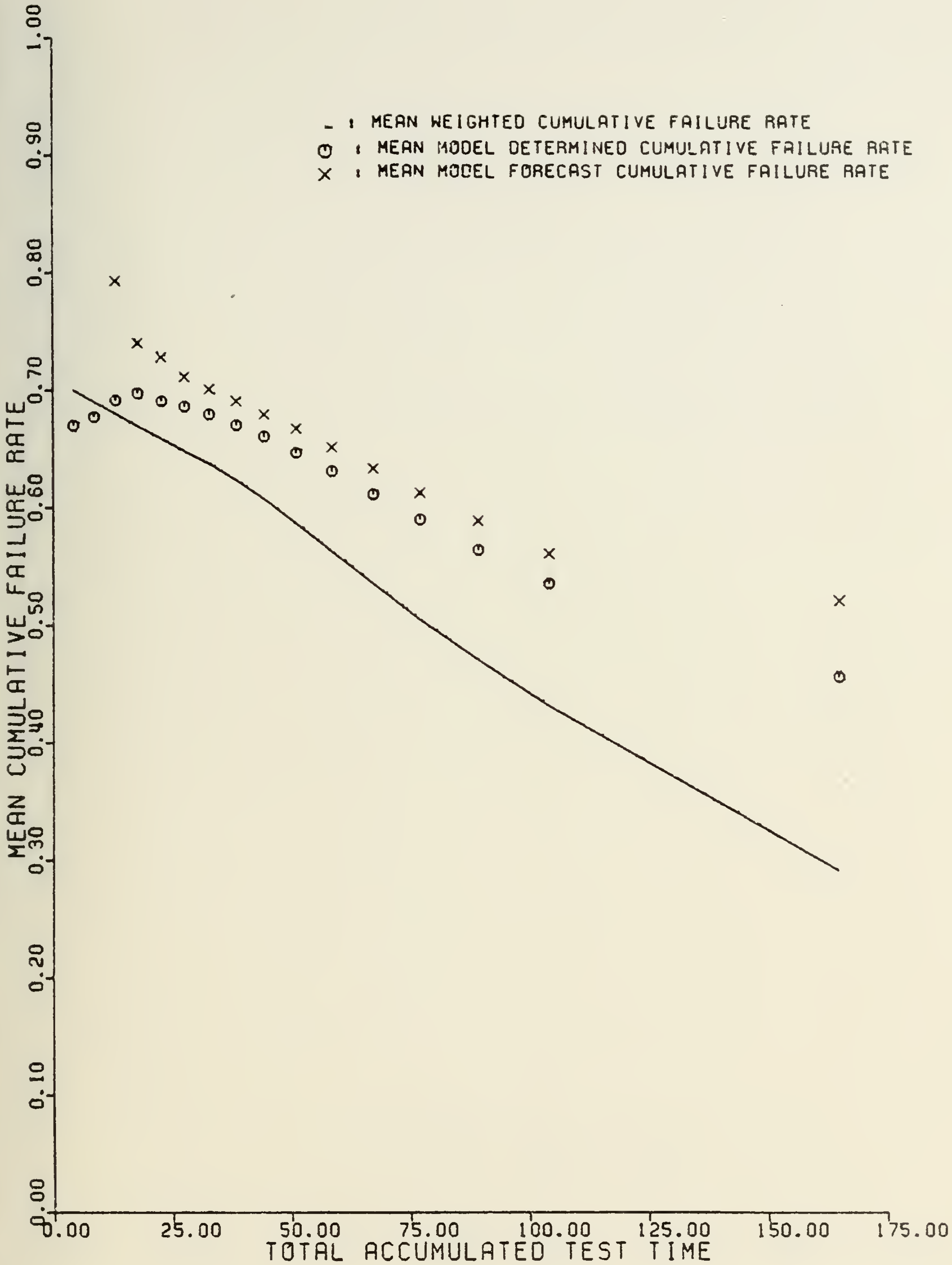

FIGURE 3.12

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 7: 16 PHASES, 10 TESTS/PHASE

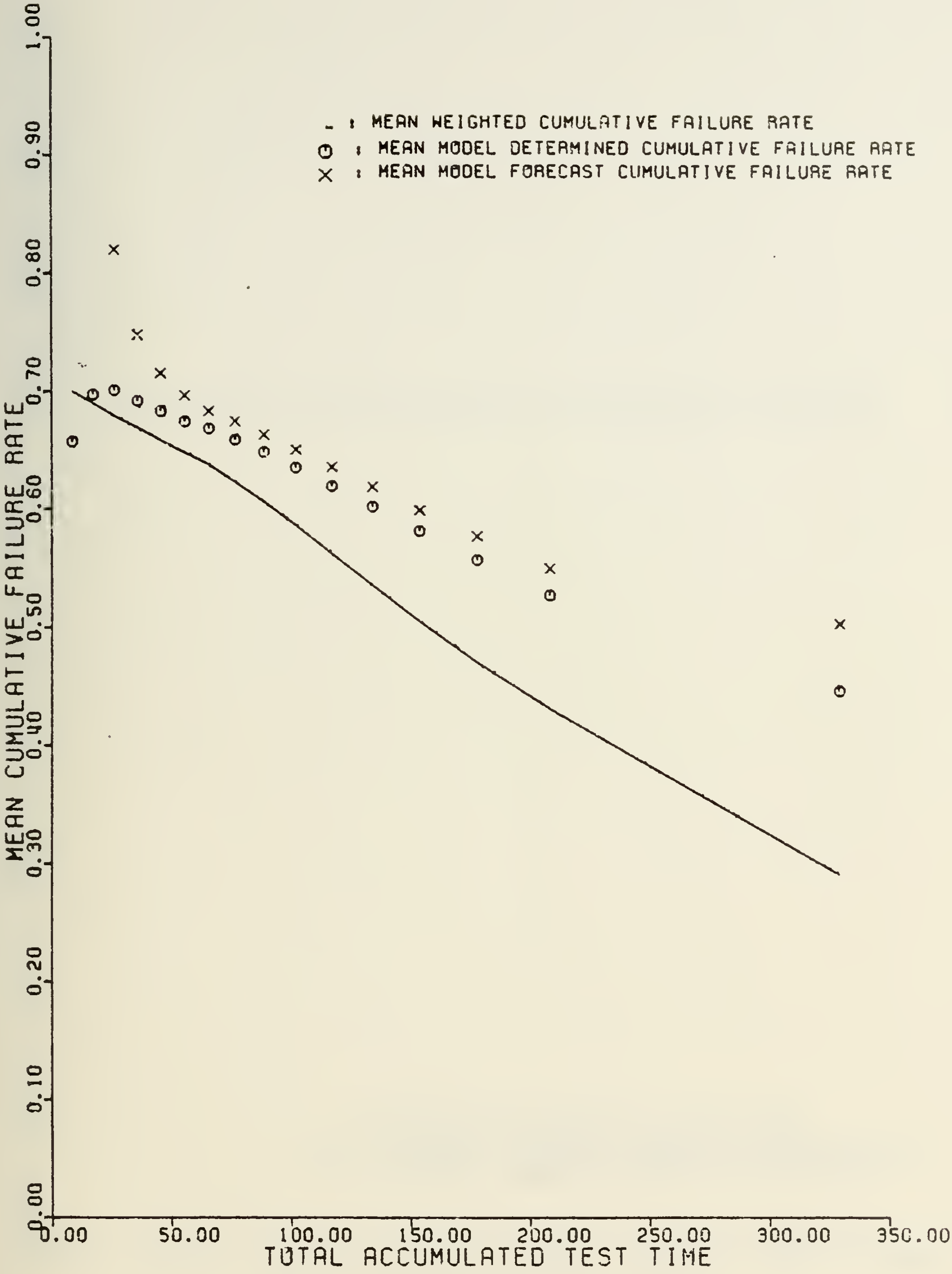

FIGURE 3.13

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 7: 16 PHASES, 20 TESTS/PHASE

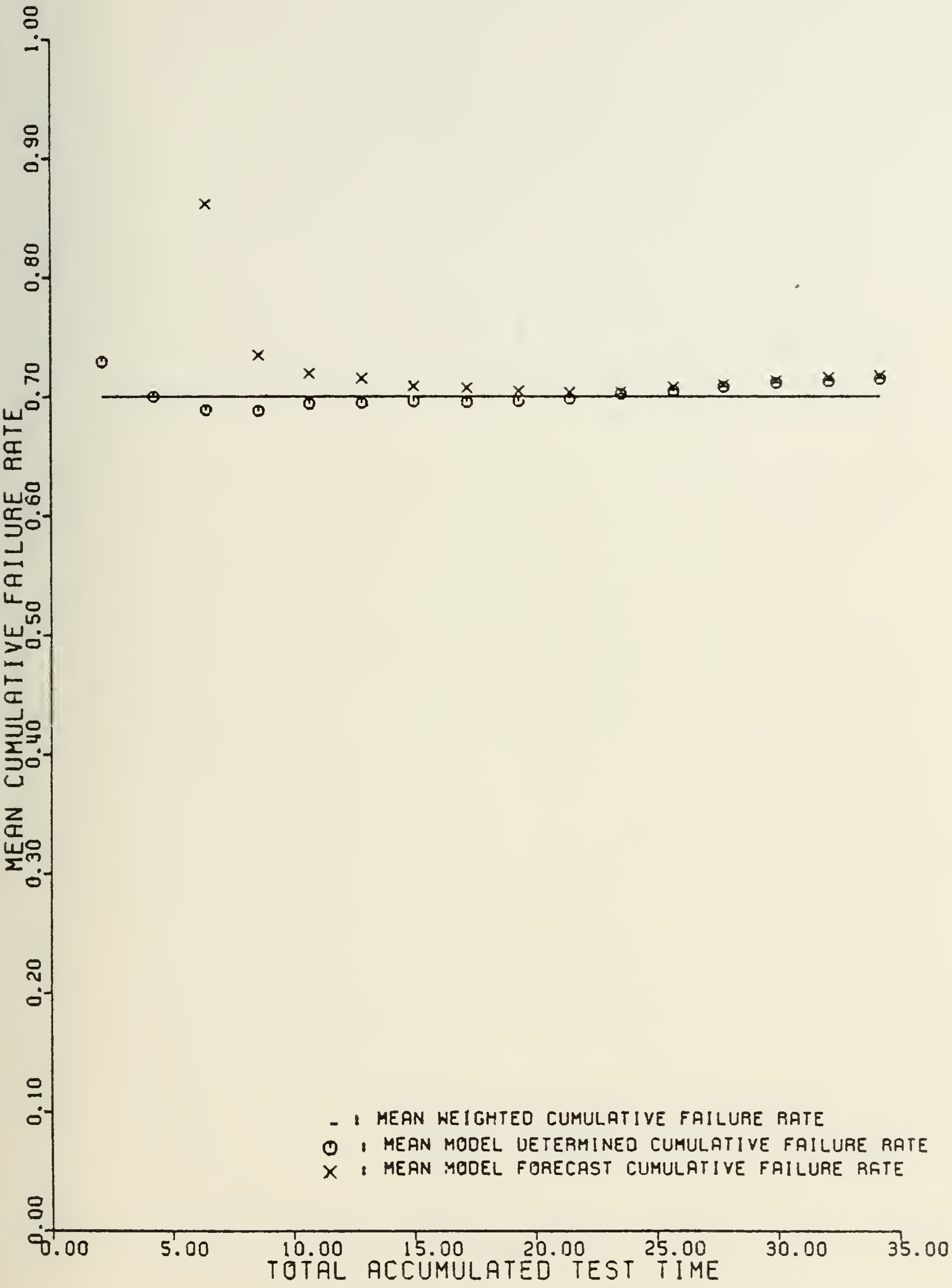

FIGURE 3.14

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 8: 16 PHASES, 5 TESTS/PHASE

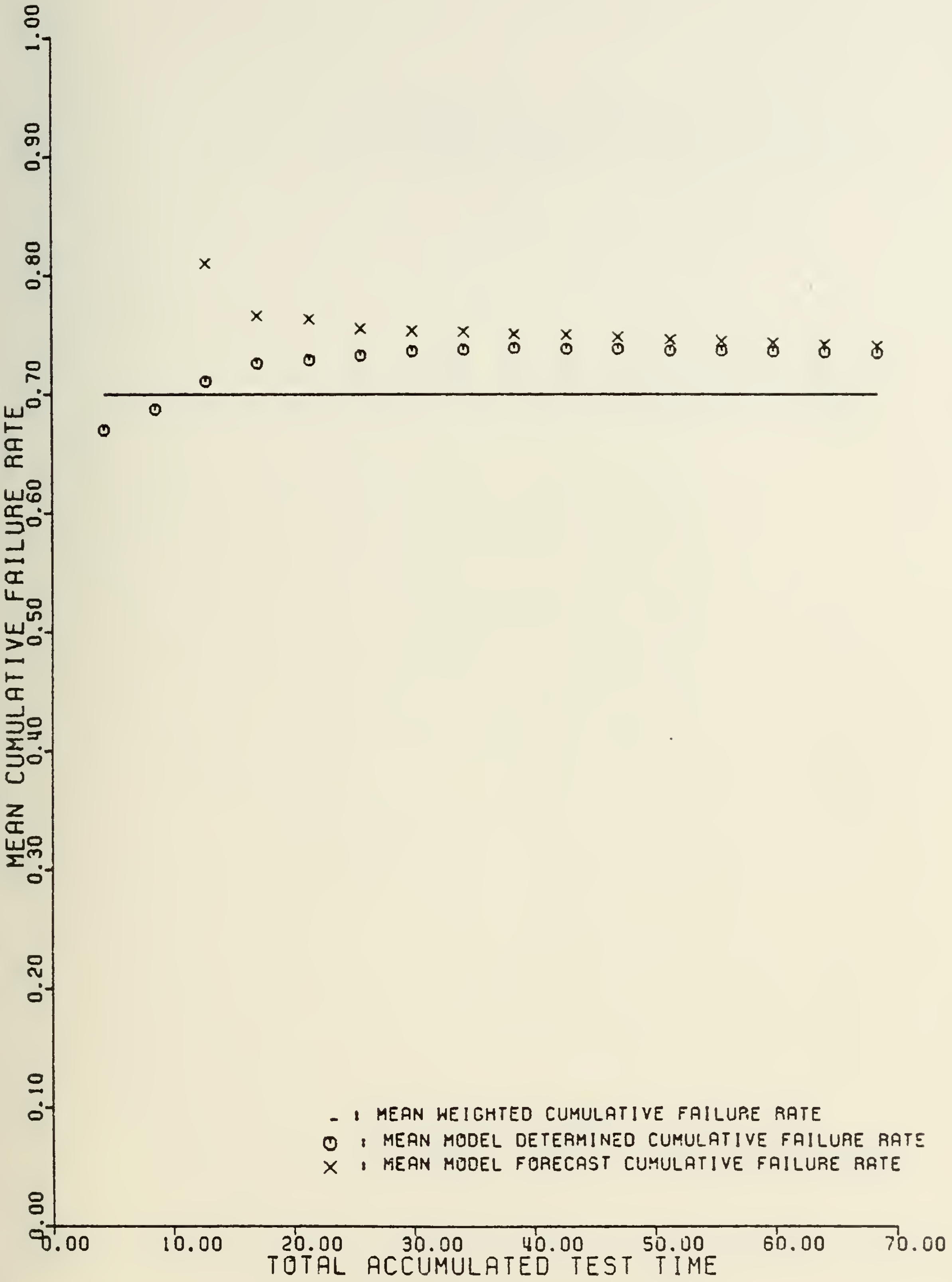

FIGURE 3.15

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 8: 16 PHASES, 10 TESTS/PHASE

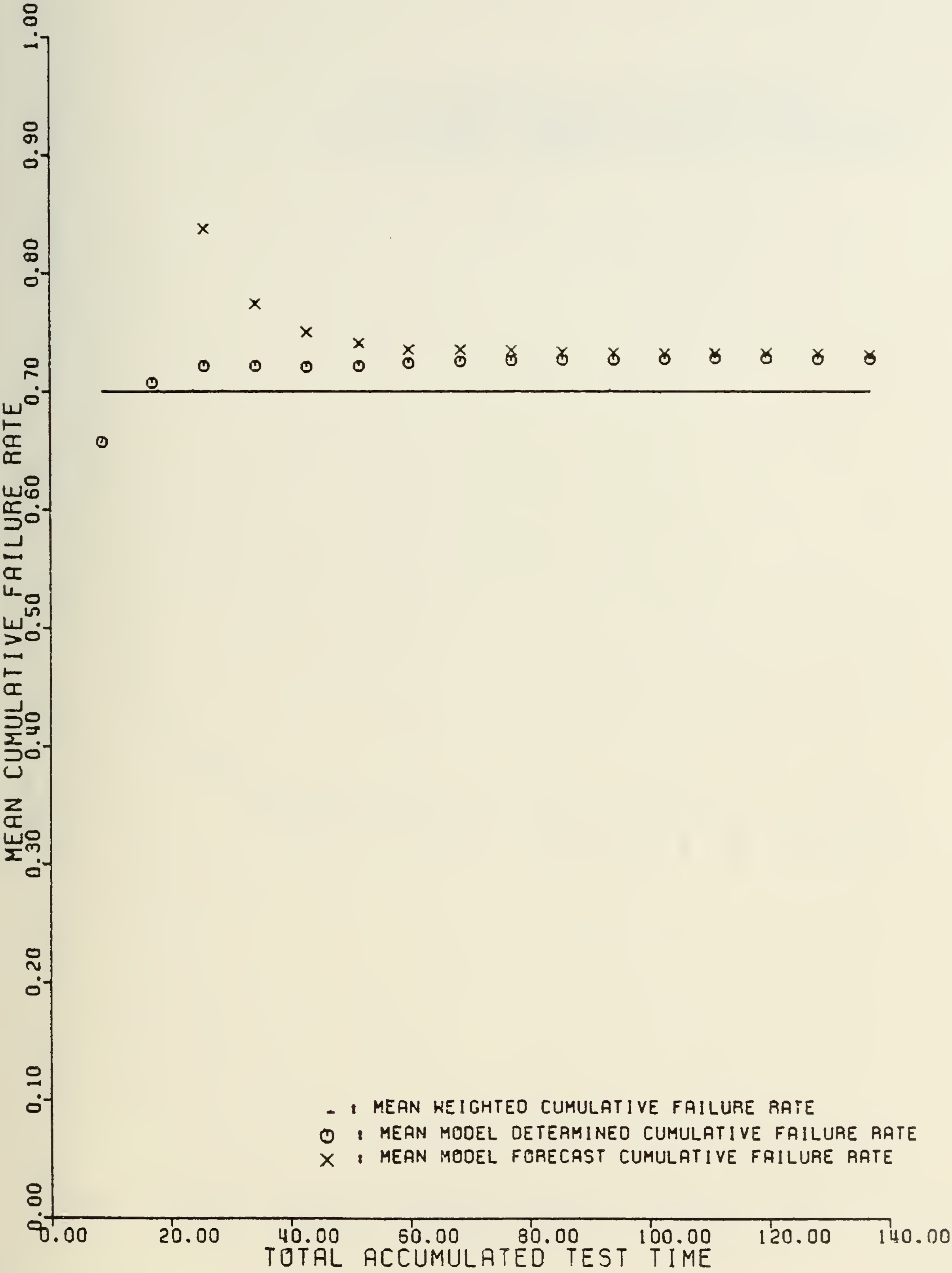

FIGURE 3.16

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 8: 16 PHASES, 20 TESTS/PHASE

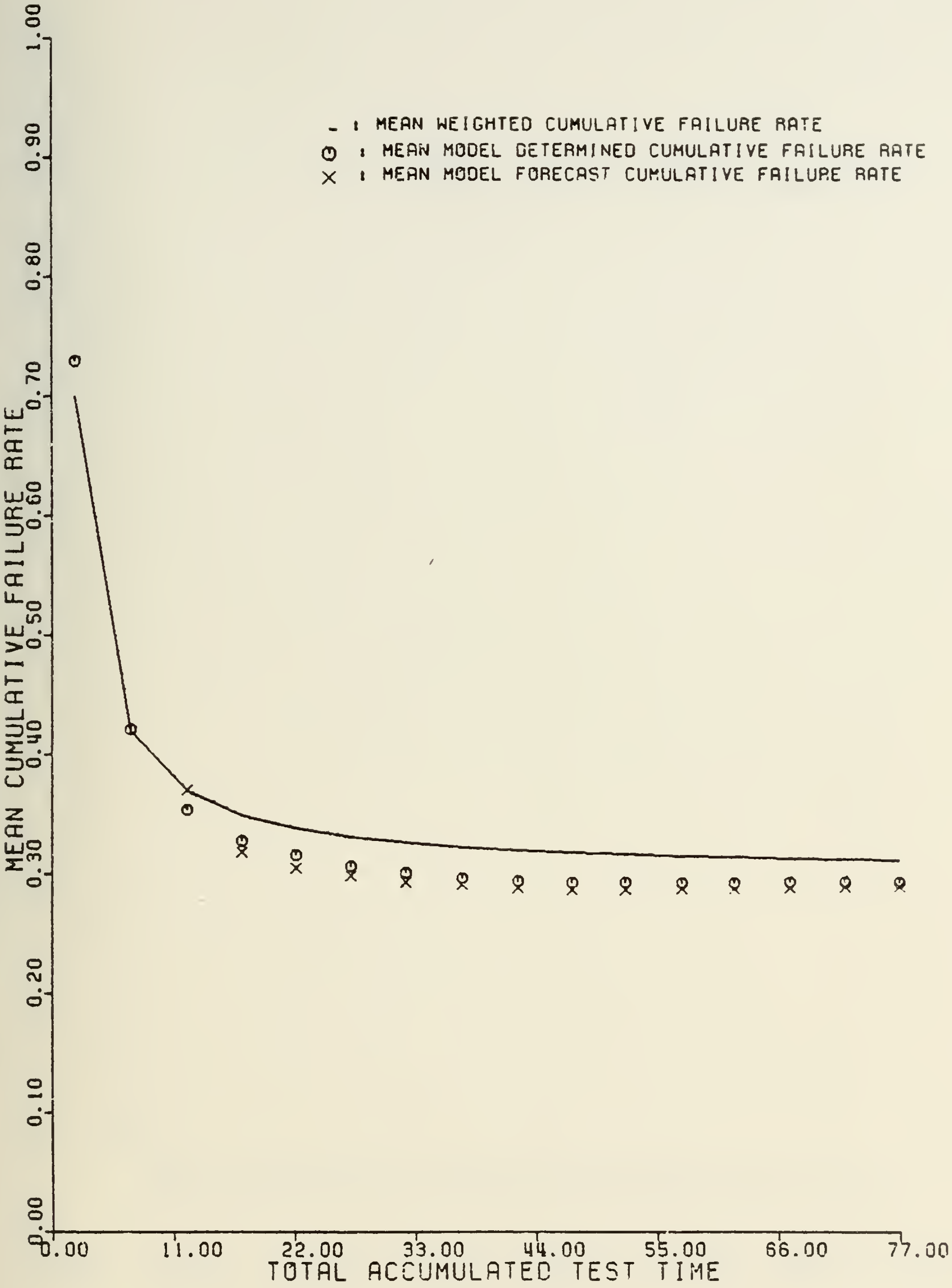

FIGURE 3.17

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 9: 16 PHASES, 5 TESTS/PHASE

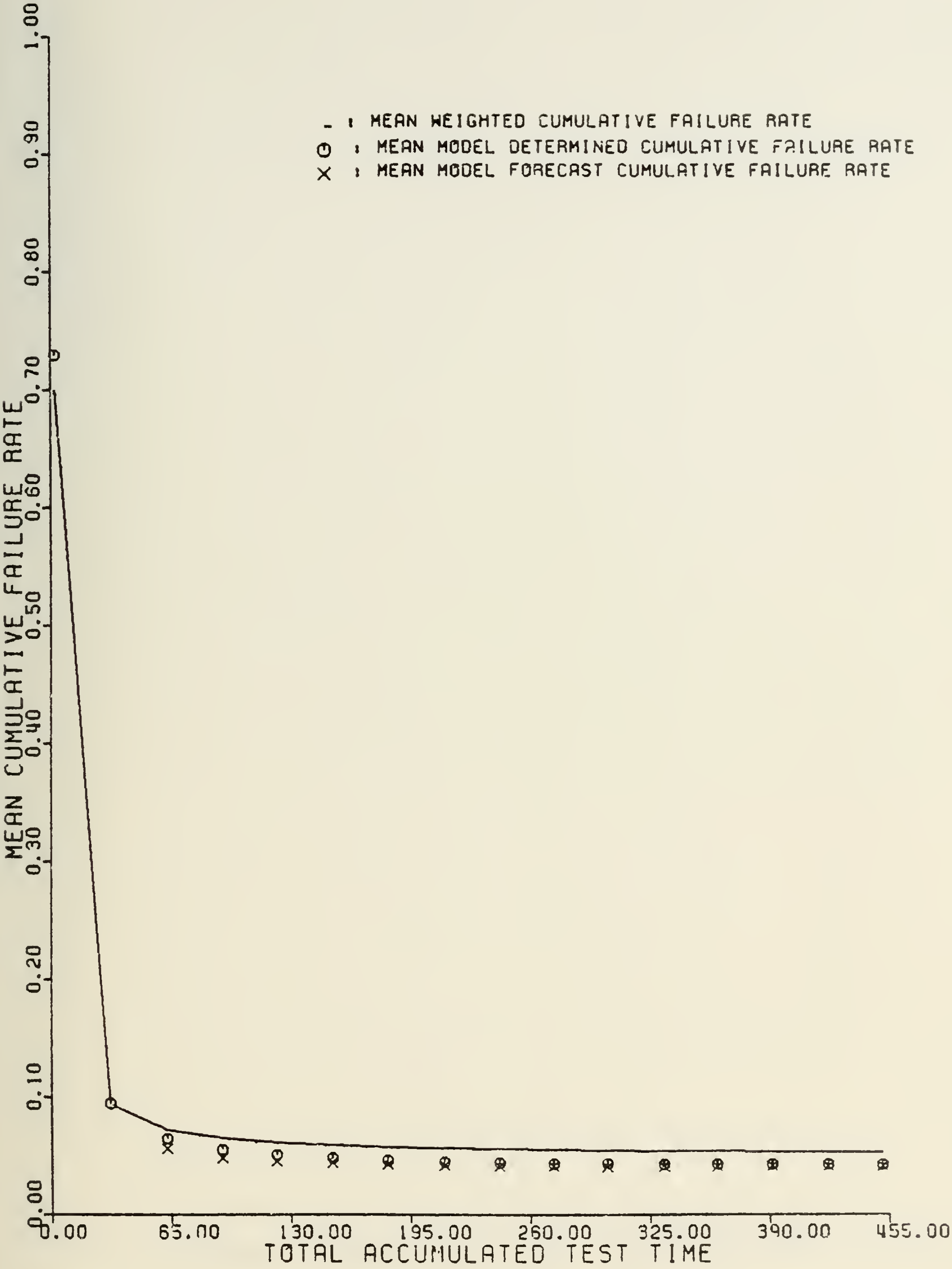

FIGURE 3.18

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE

LAMBDA SET 10: 16 PHASES, 5 TESTS/PHASE

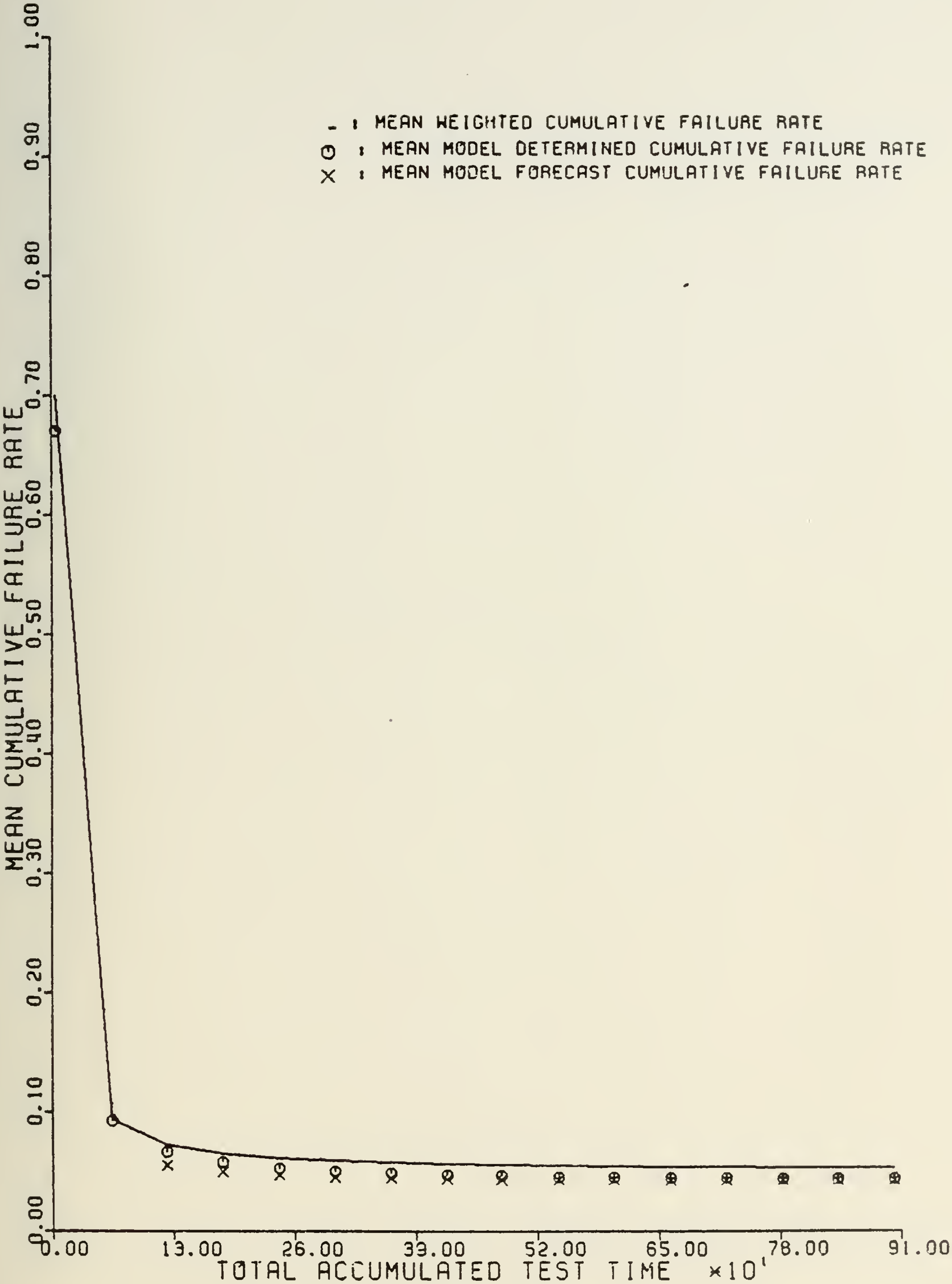

FIGURE 3.19
CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 10: 16 PHASES, 10 TESTS/PHASE

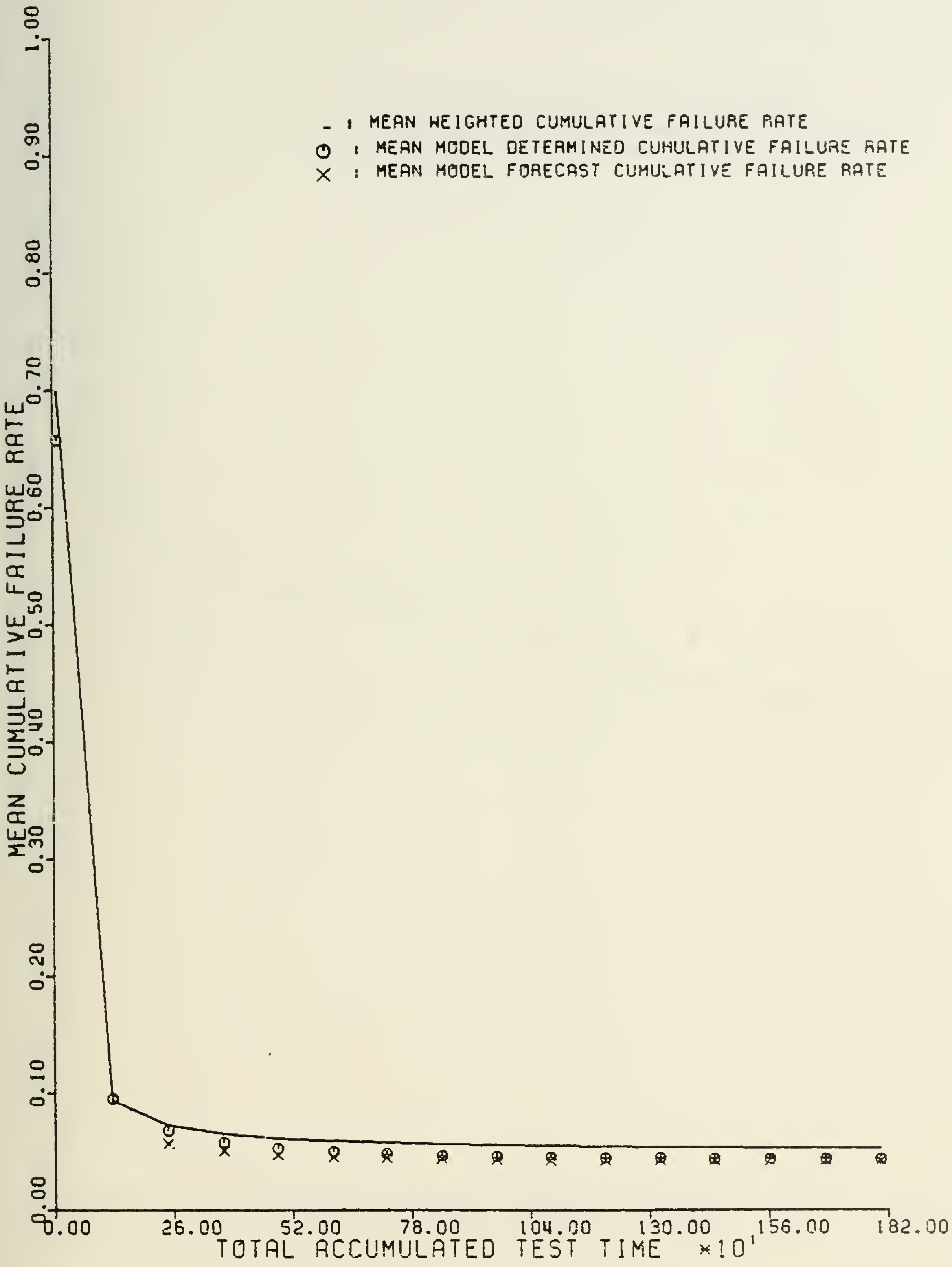

FIGURE 3.20

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 10: 16 PHASES, 20 TESTS/PHASE

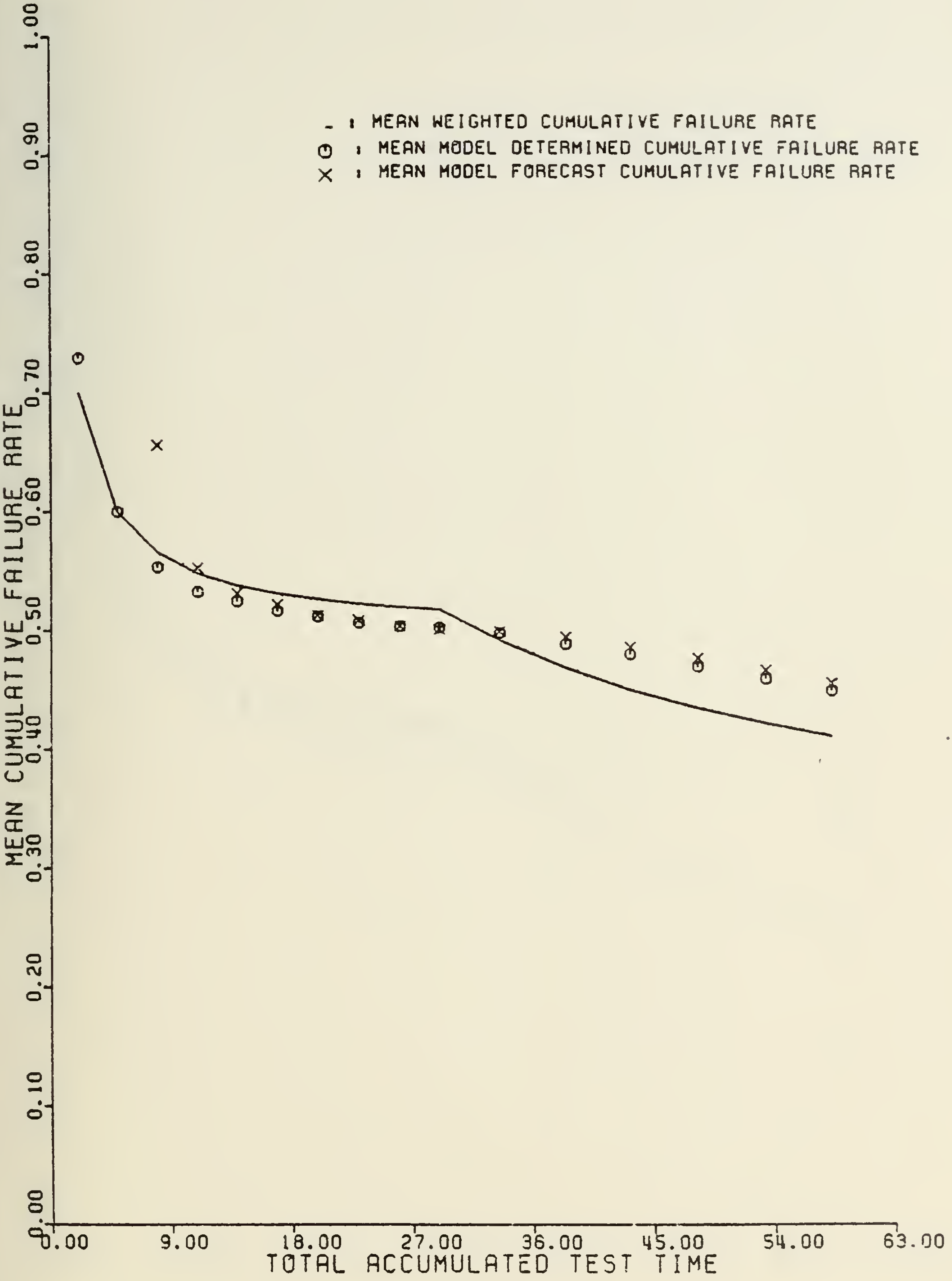

FIGURE 3.21

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 11: 16 PHASES, 5 TESTS/PHASE

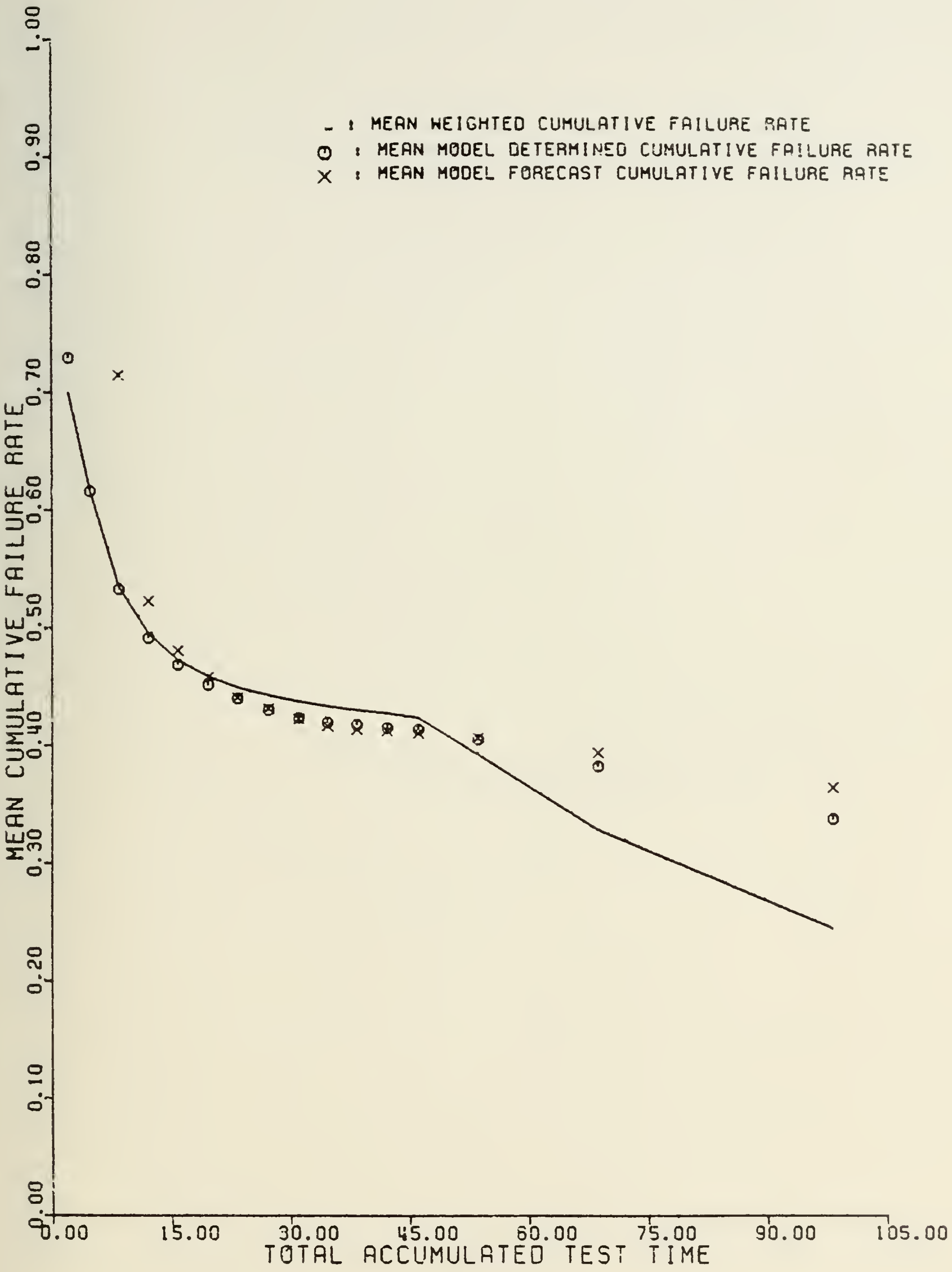

FIGURE 3.22

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE

LAMBDA SET 12: 16 PHASES, 5 TESTS/PHASE

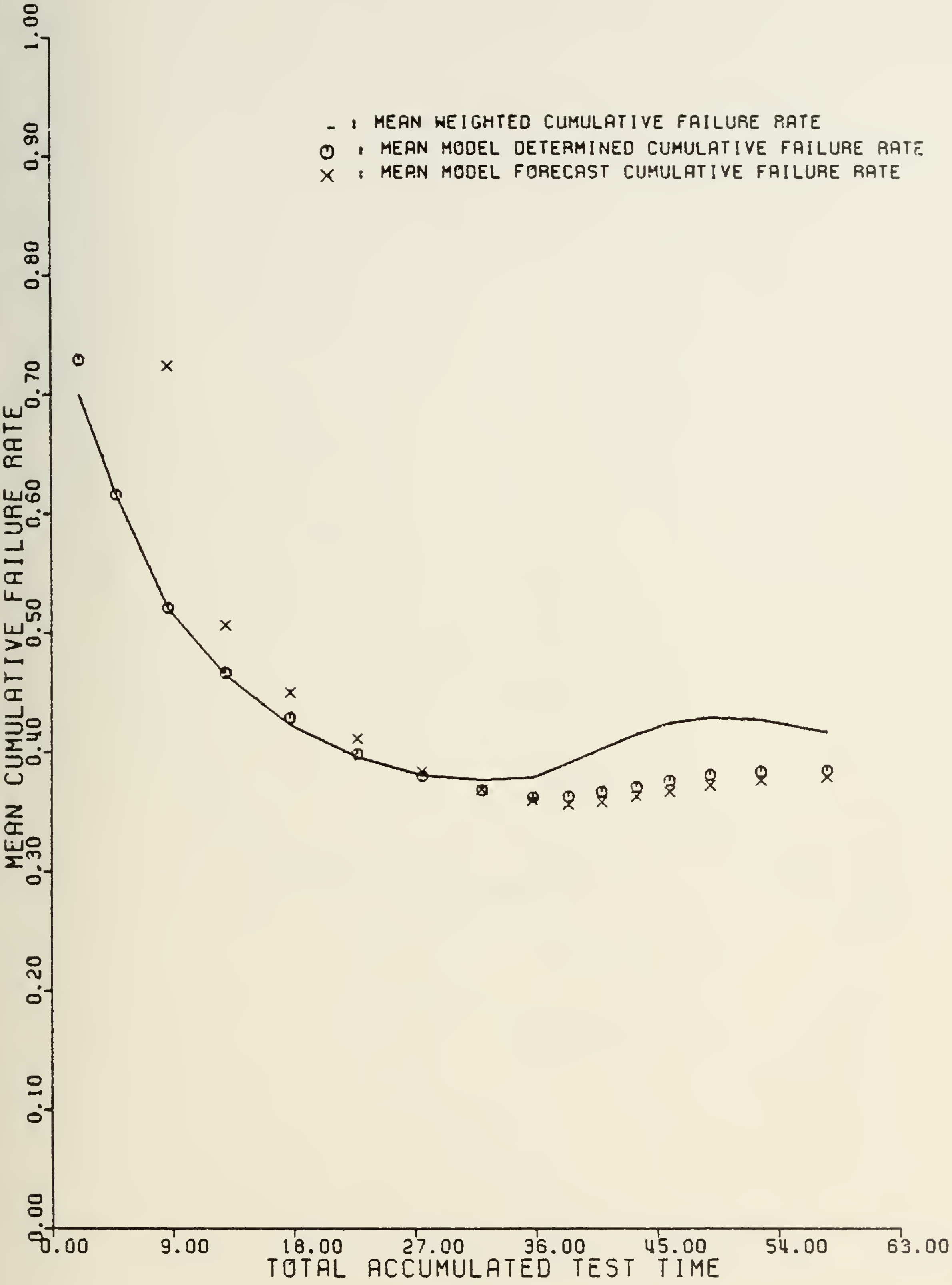

FIGURE 3.23

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 13: 16 PHASES, 5 TESTS/PHASE

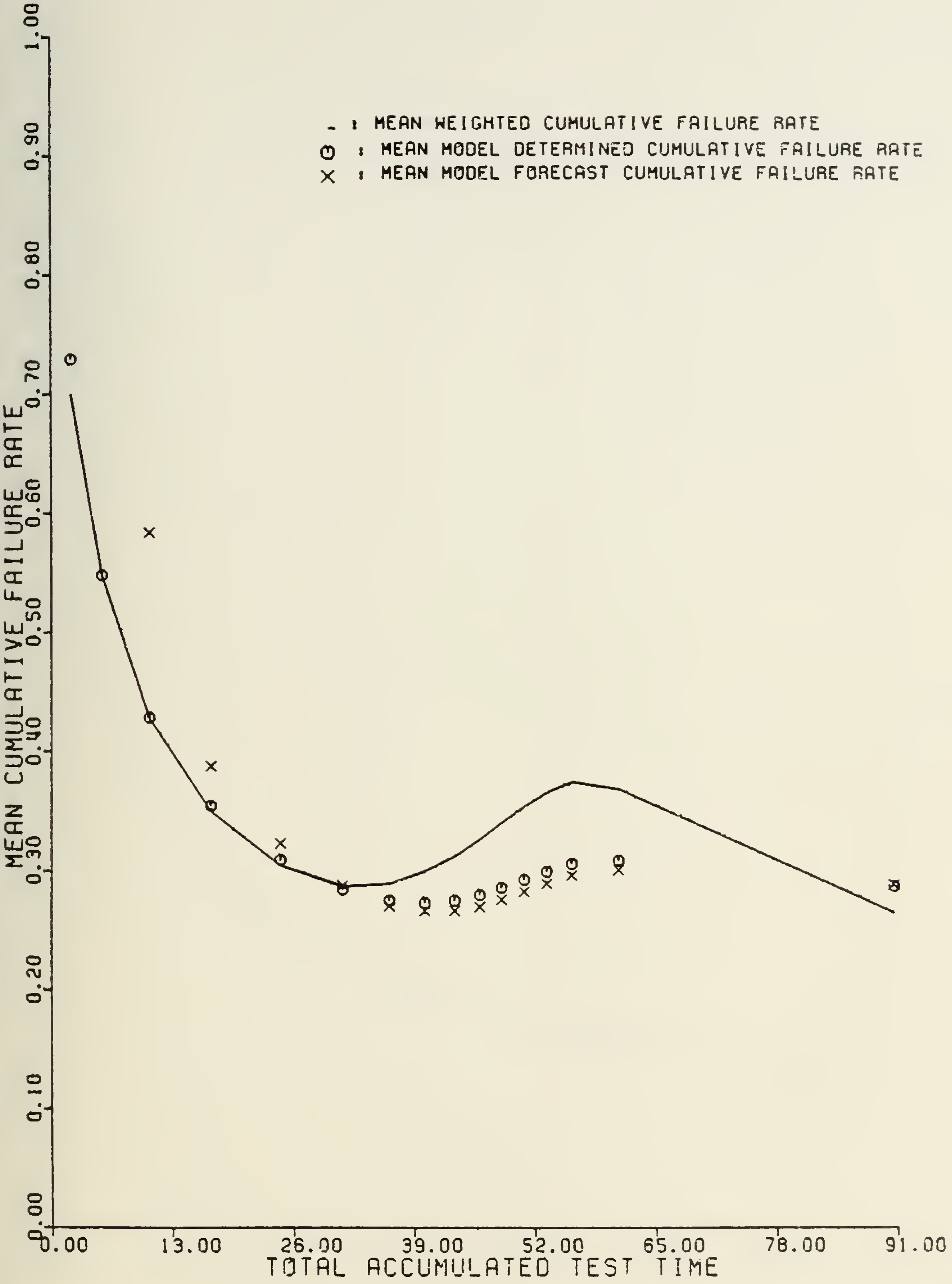

FIGURE 3.24
CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 14: 16 PHASES, 5 TESTS/PHASE

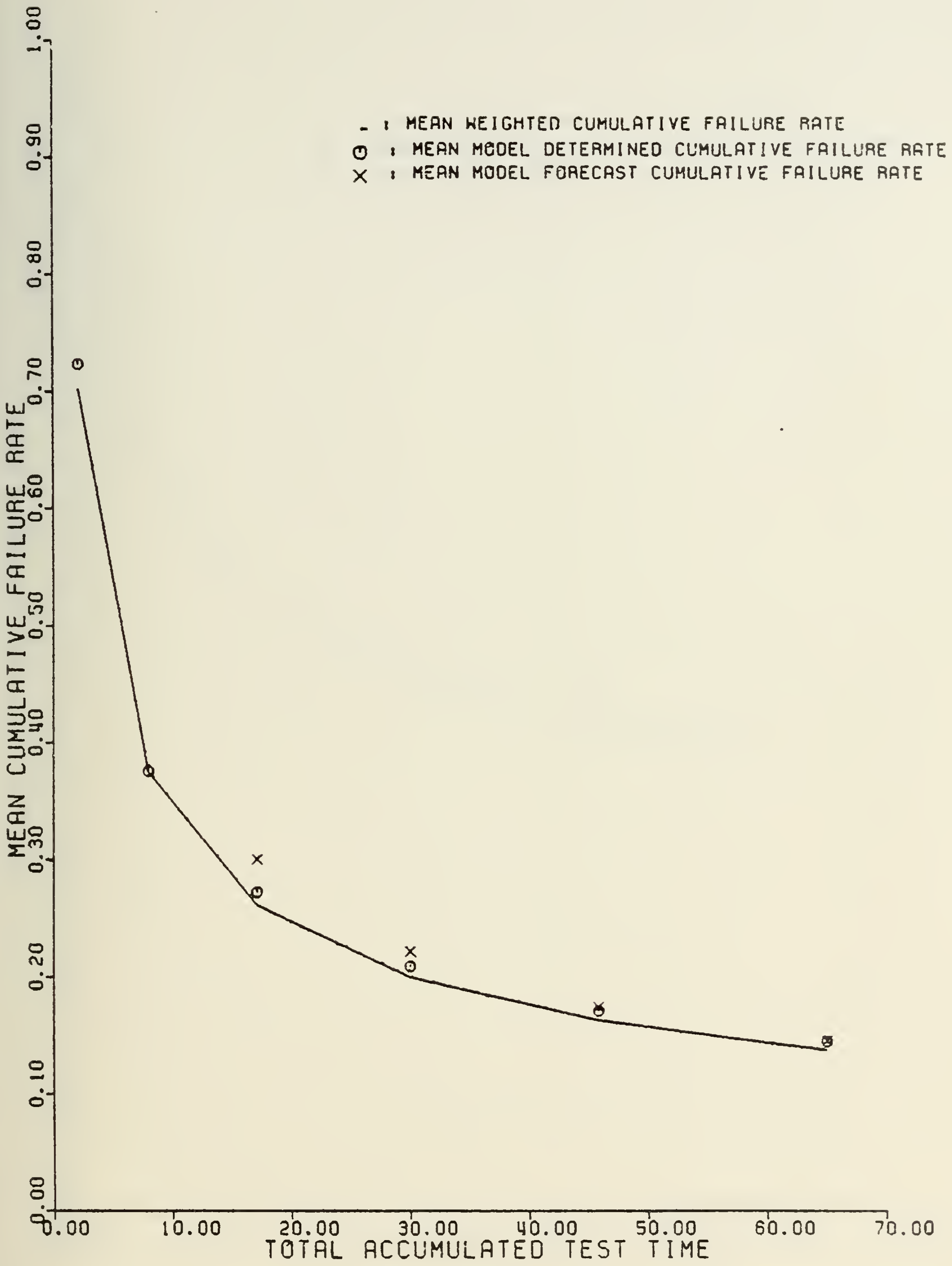

FIGURE 3.25

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD1: 6 PHASES, 5 TESTS/PHASE

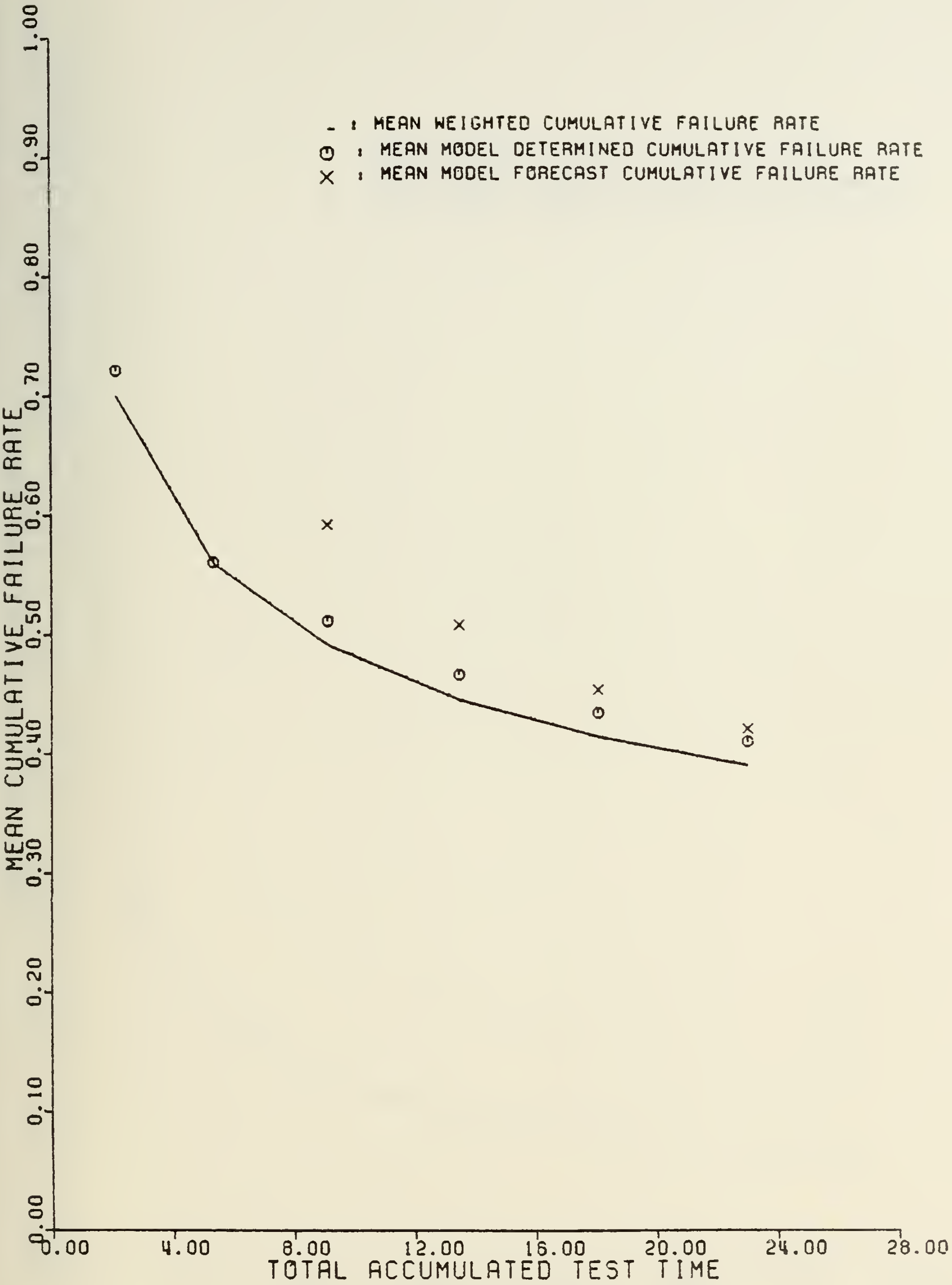

FIGURE 3.26

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD2: 6 PHASES, 5 TESTS/PHASE

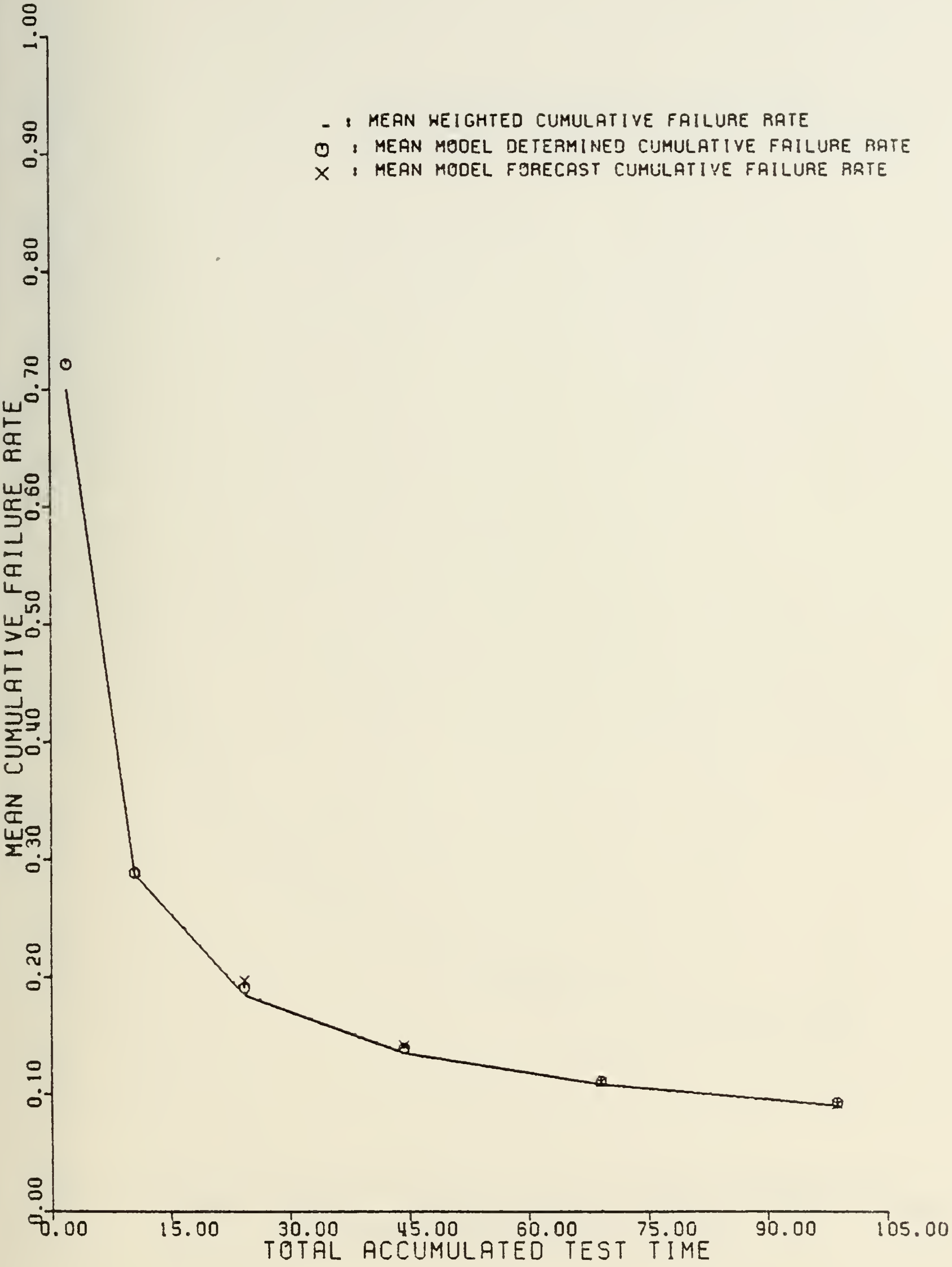

FIGURE 3.27
CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD3: 6 PHASES, 5 TESTS/PHASE

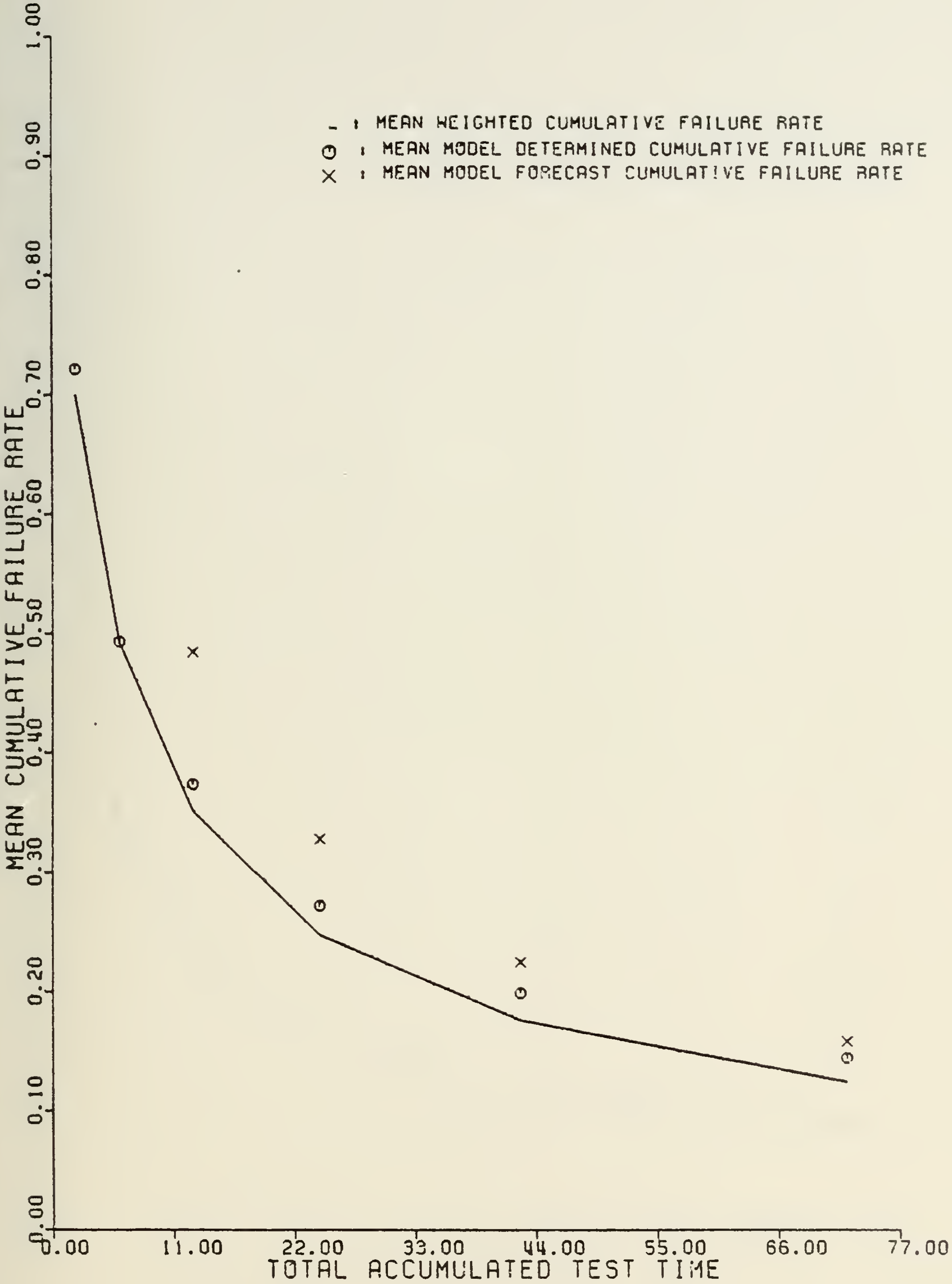

FIGURE 3.28

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD4: 6 PHASES, 5 TESTS/PHASE

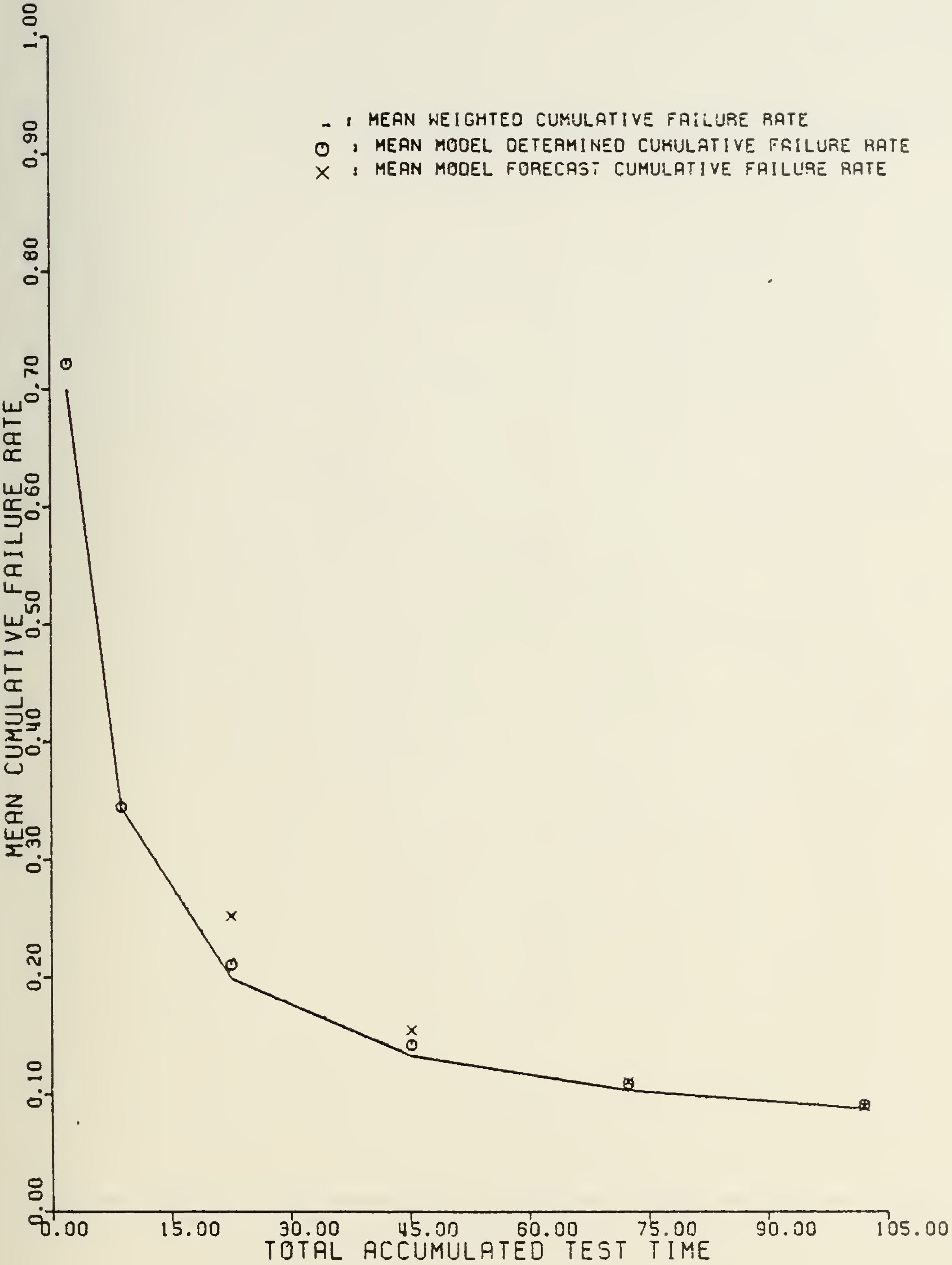

FIGURE 3.29
CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MODS: 6 PHASES, 5 TESTS/PHASE

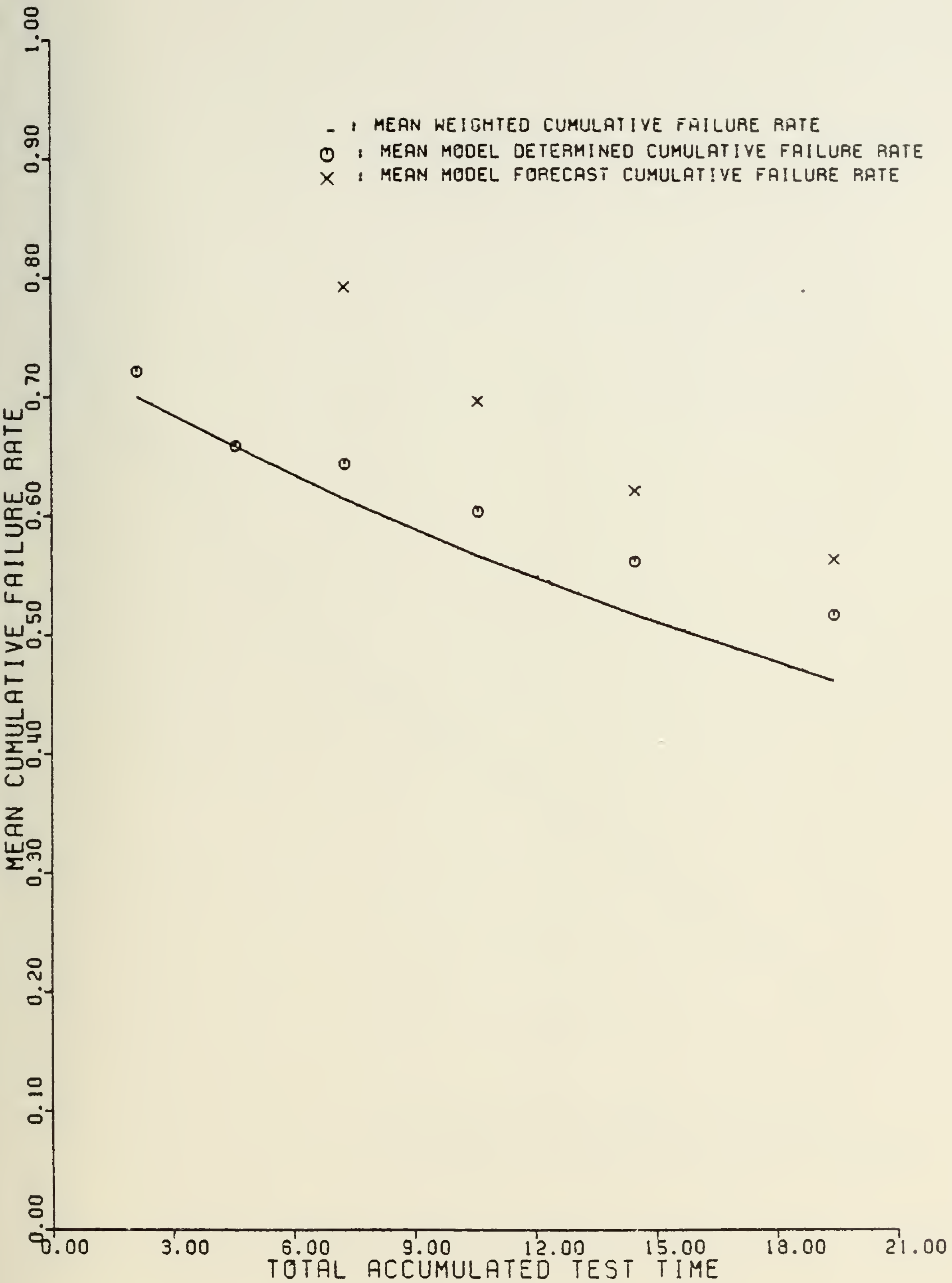

FIGURE 3.30

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD6: 6 PHASES, 5 TESTS/PHASE

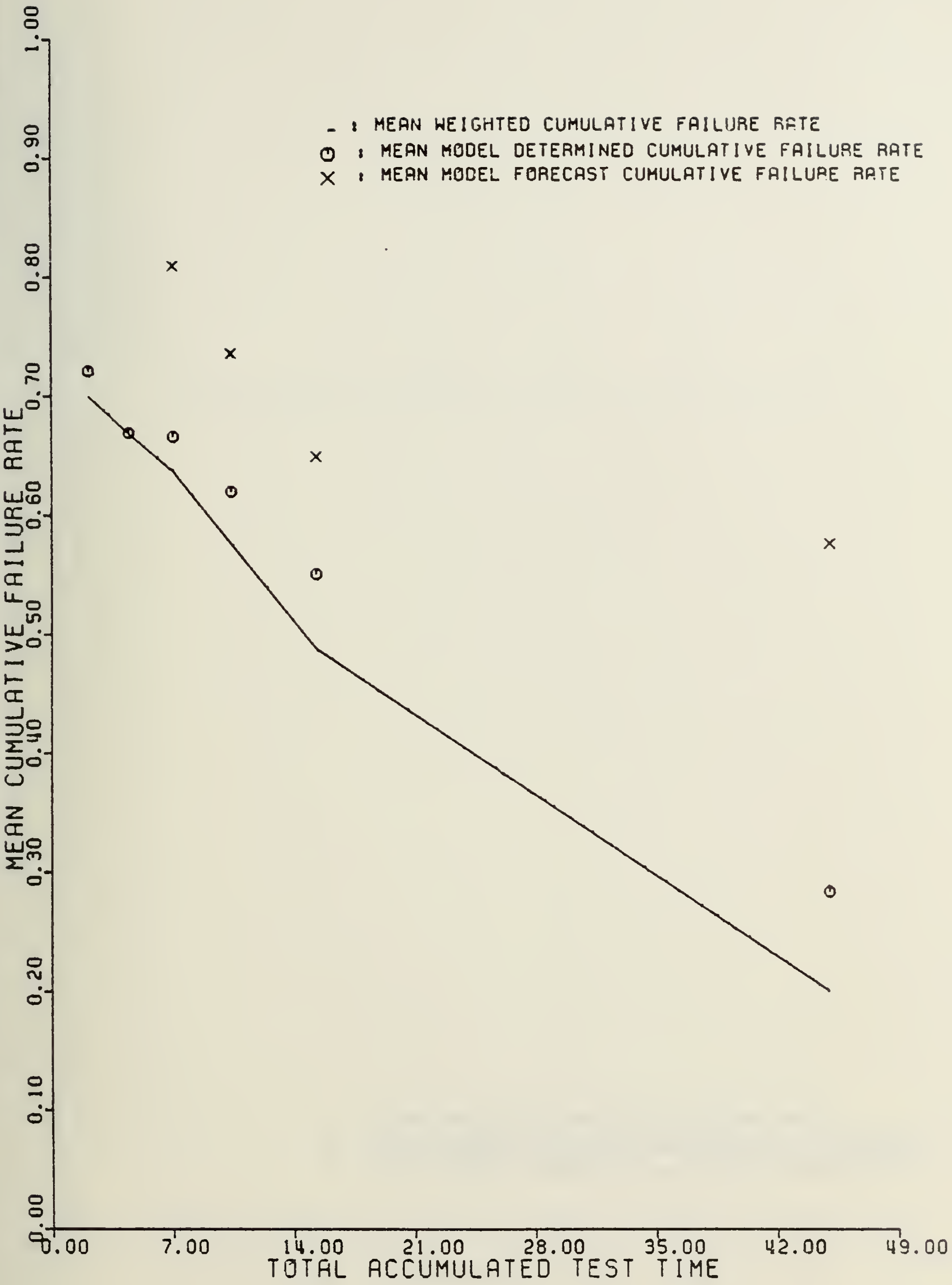

FIGURE 3.31

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD7: 6 PHASES, 5 TESTS/PHASE

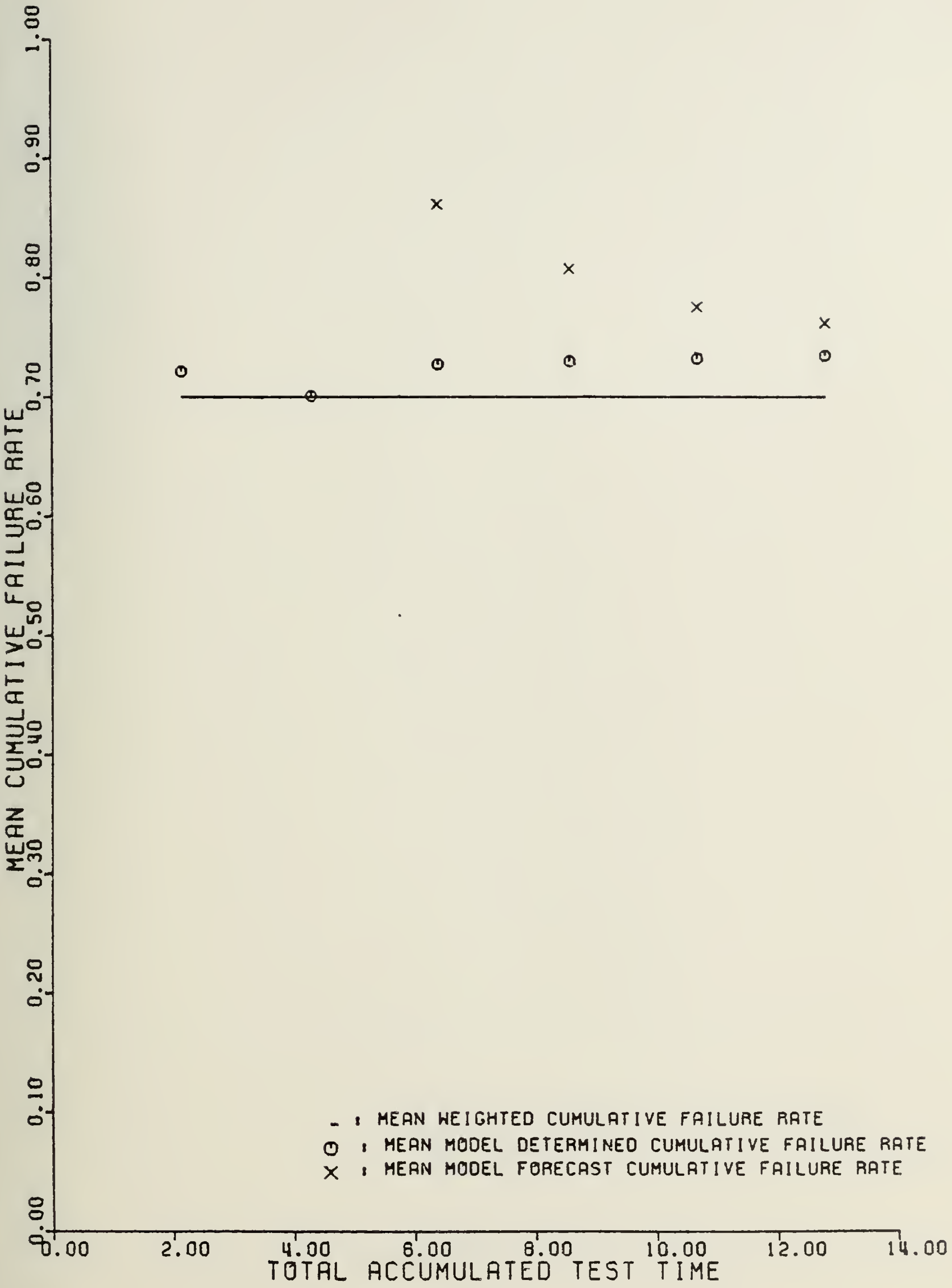

FIGURE 3.32

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD8: 6 PHASES, 5 TESTS/PHASE

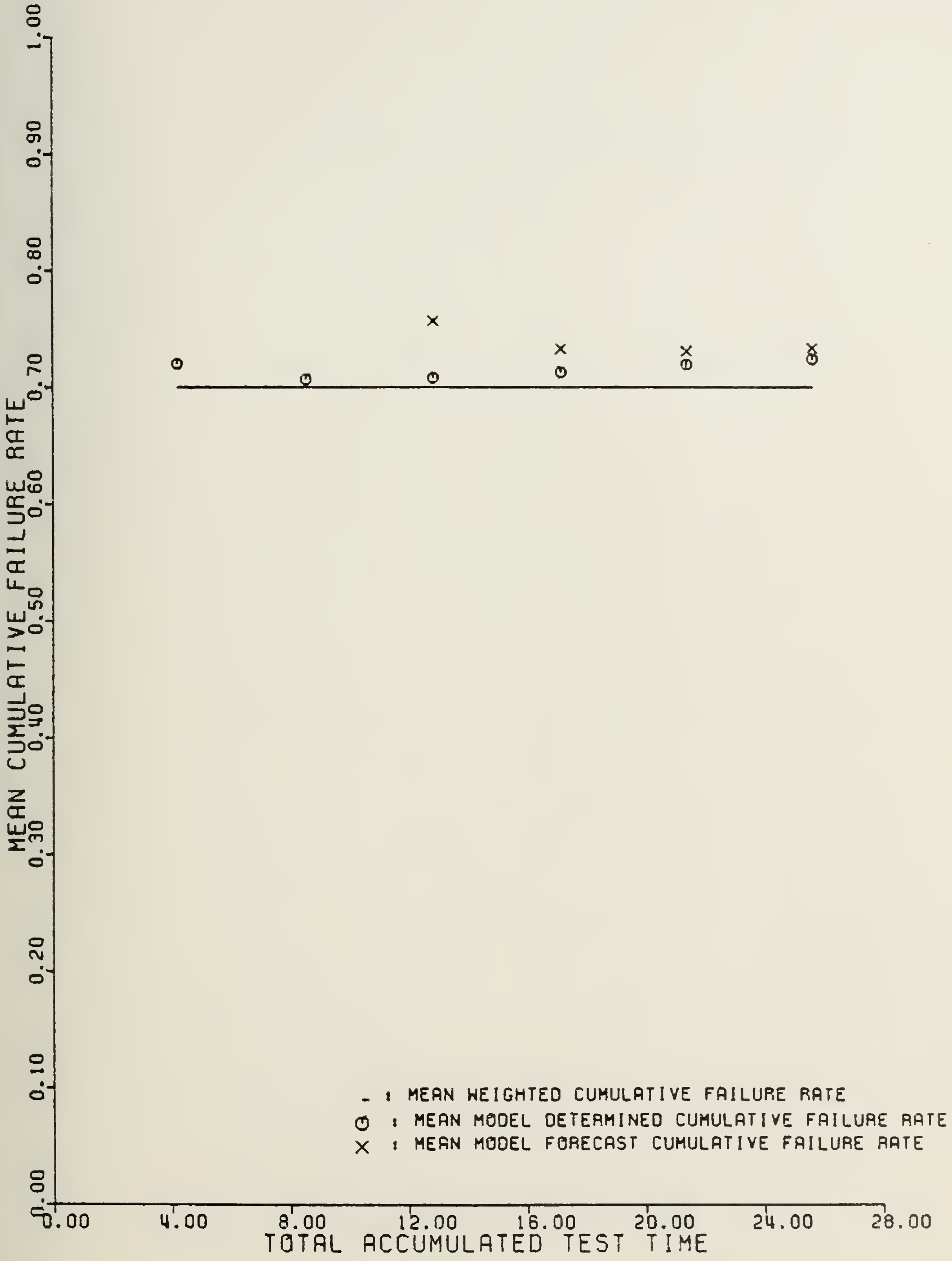

FIGURE 3.33

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD8: 6 PHASES, 10 TESTS/PHASE

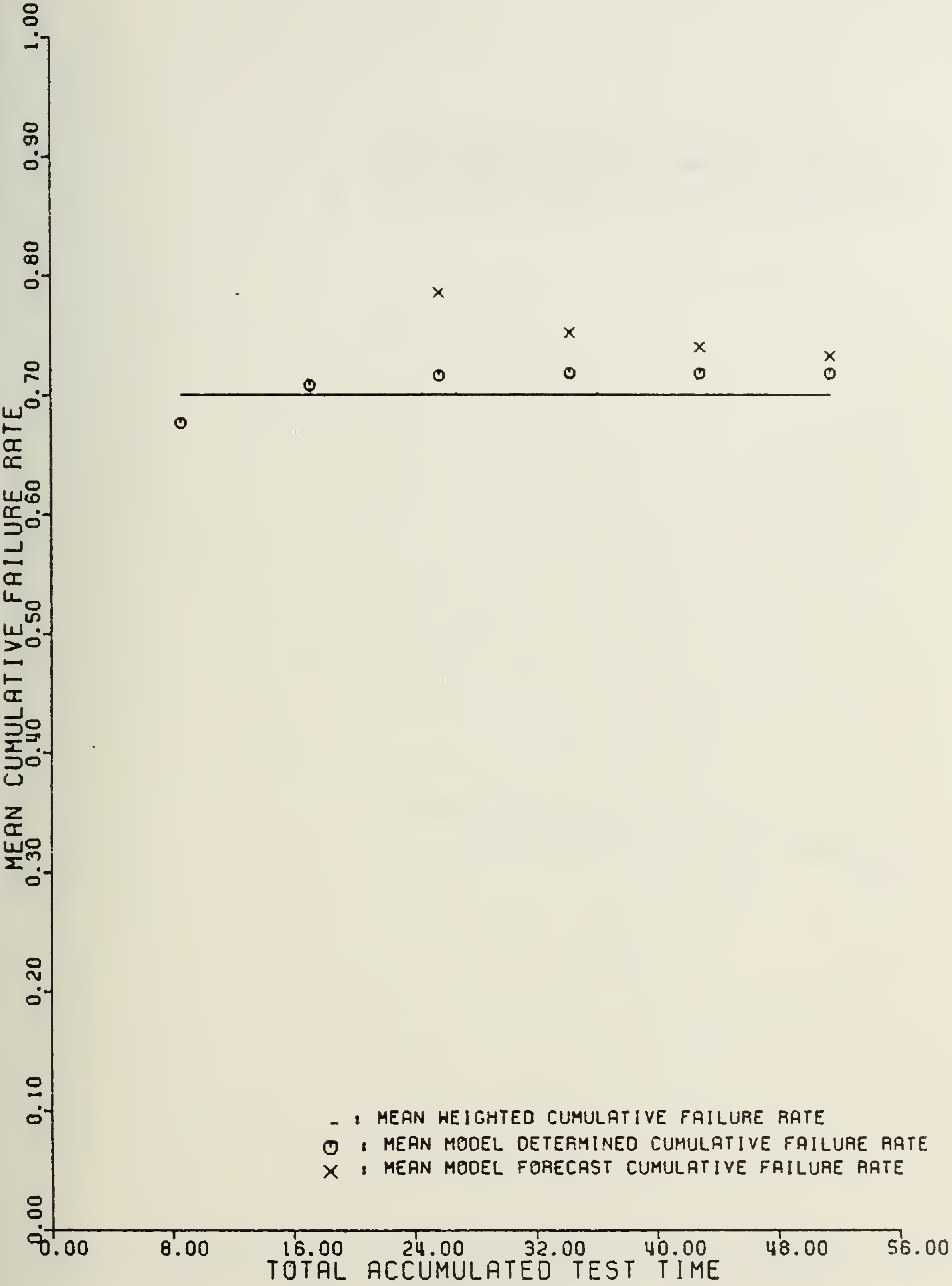

FIGURE 3.34

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD8: 6 PHASES, 20 TESTS/PHASE

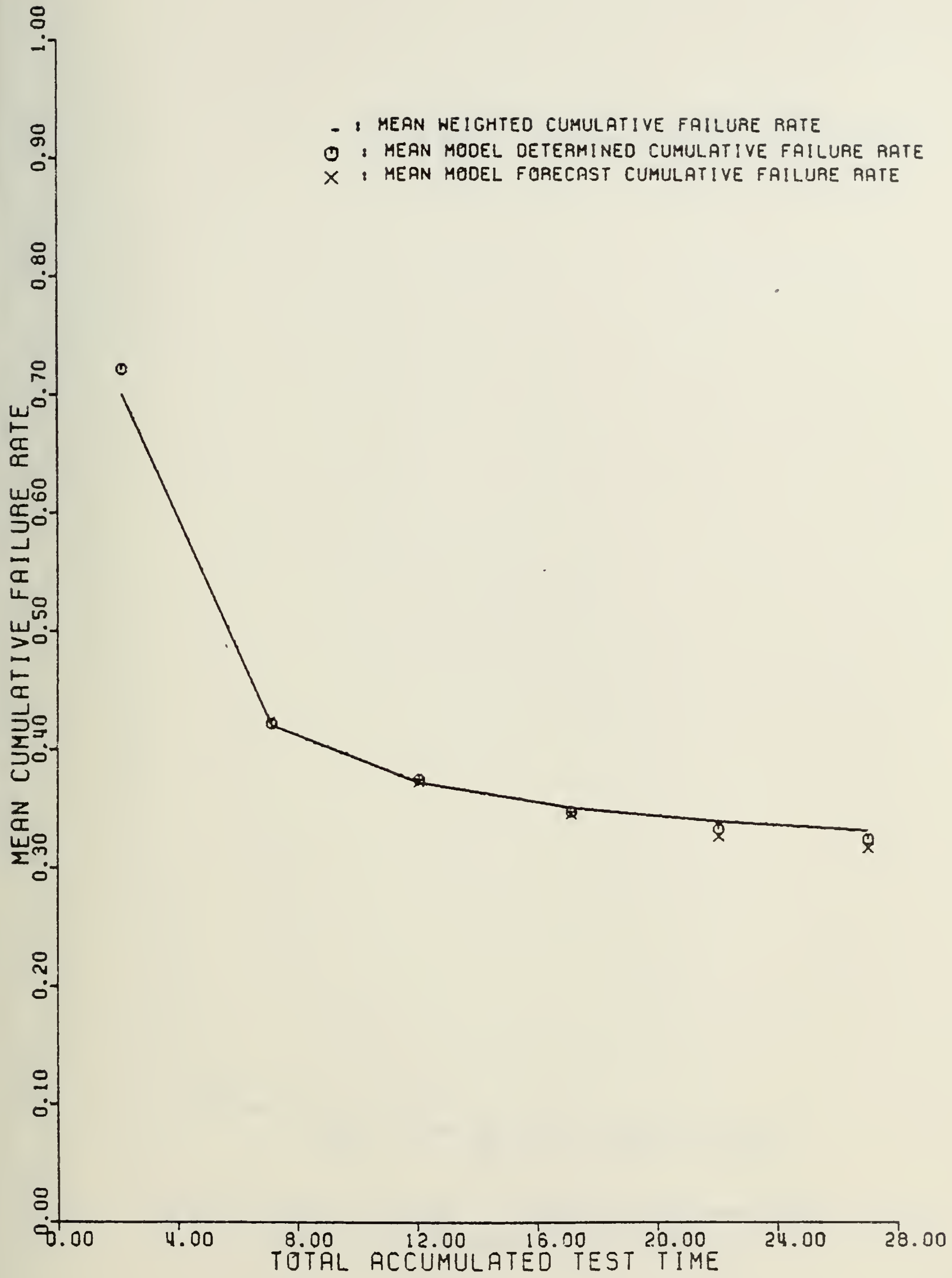

FIGURE 3.35

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD9: 6 PHASES, 5 TESTS/PHASE

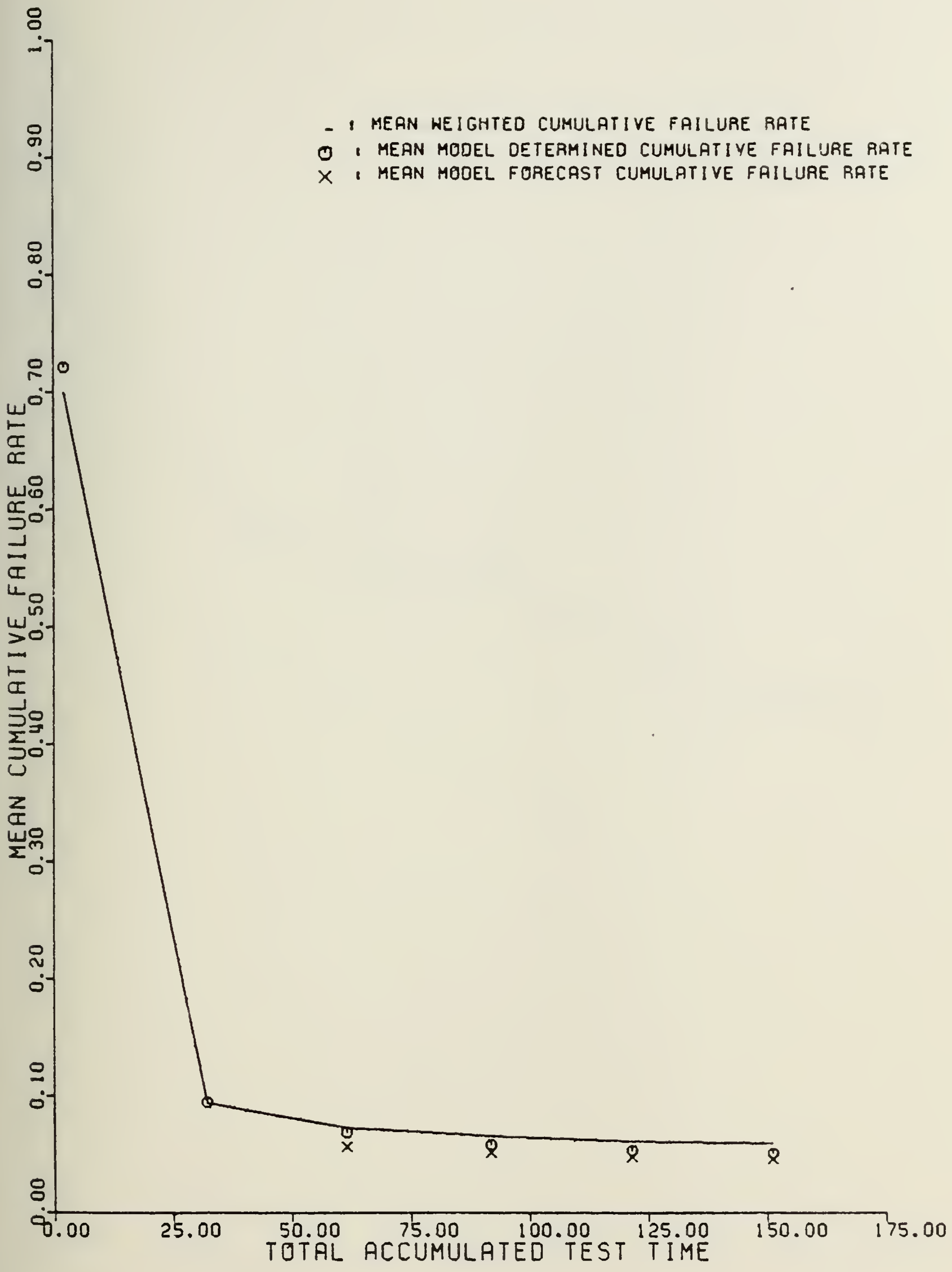

FIGURE 3.36

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD10: 6 PHASES, 5 TESTS/PHASE

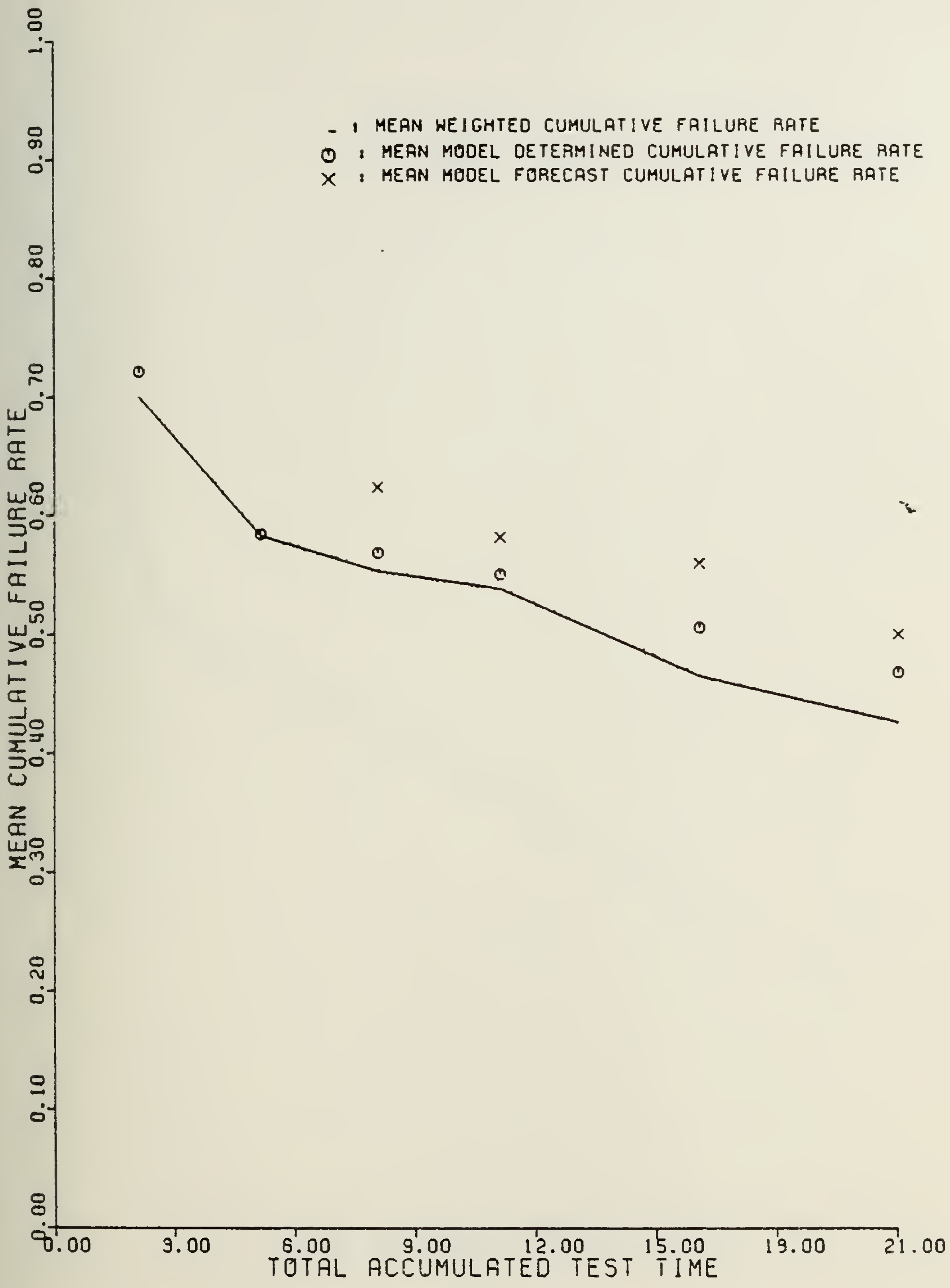

FIGURE 3.37

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD11: 6 PHASES, 5 TESTS/PHASE

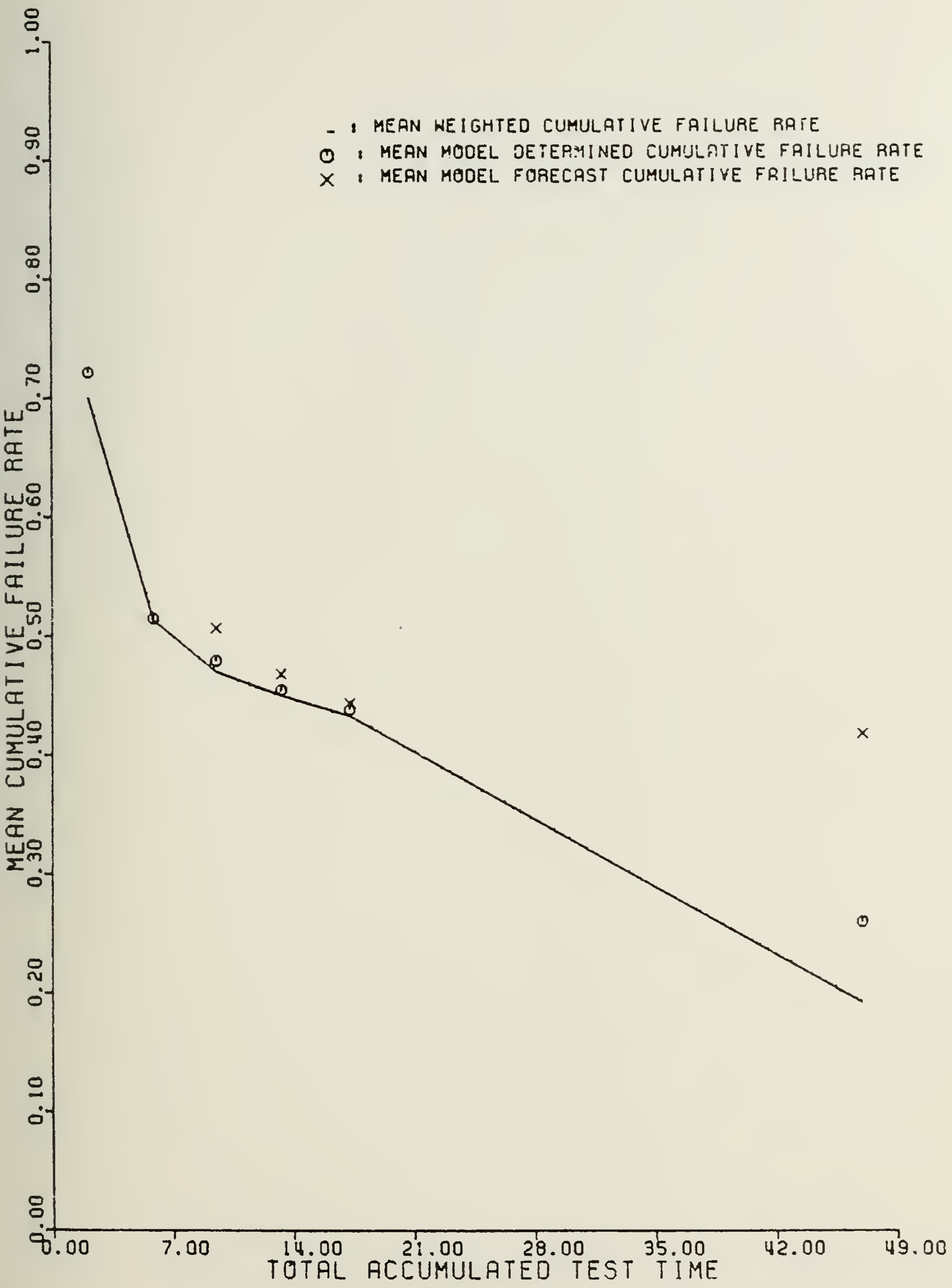

FIGURE 3.38

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD12: 6 PHASES, 5 TESTS/PHASE

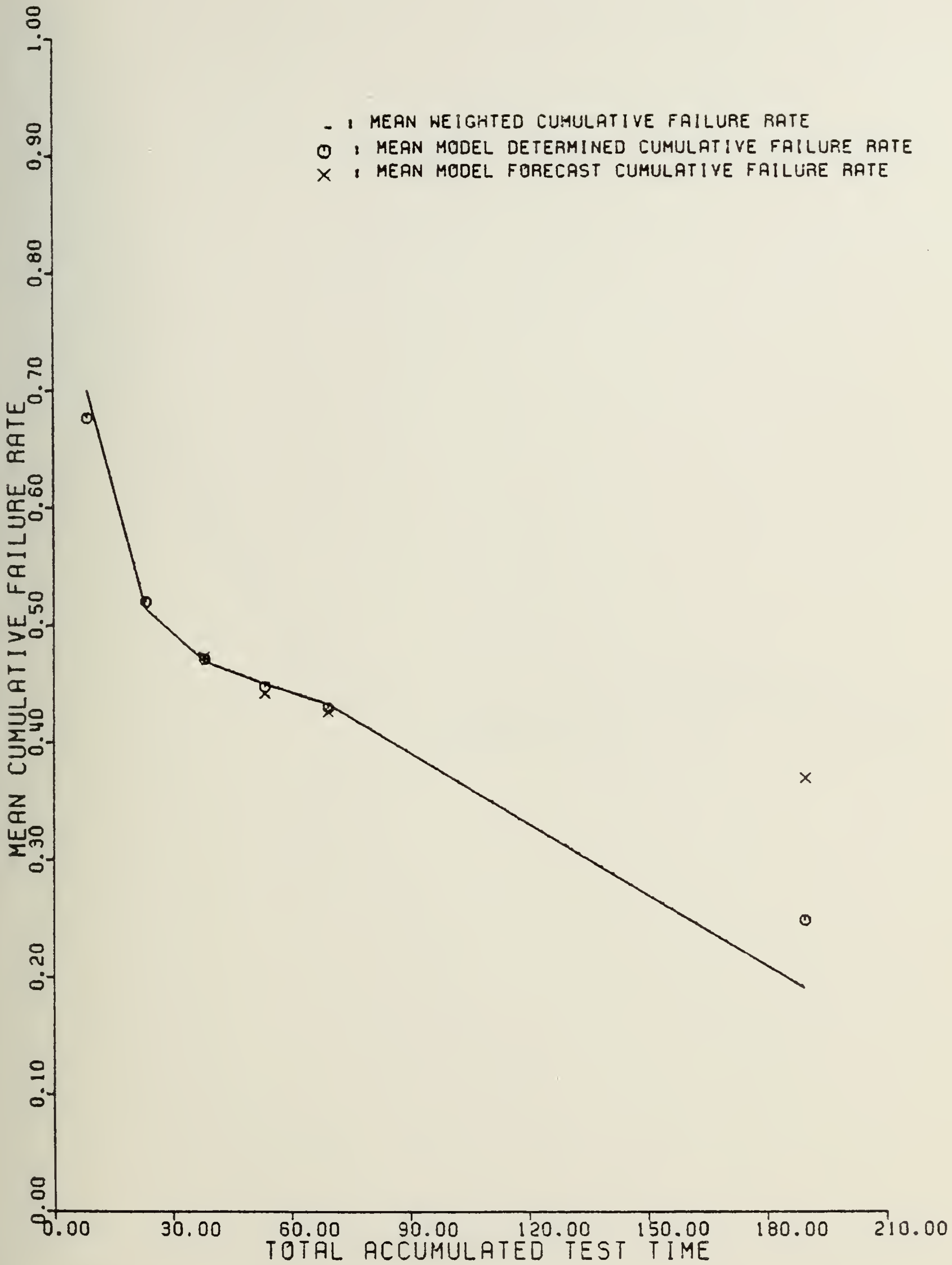

FIGURE 3.39

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD12: 6 PHASES, 20 TESTS/PHASE

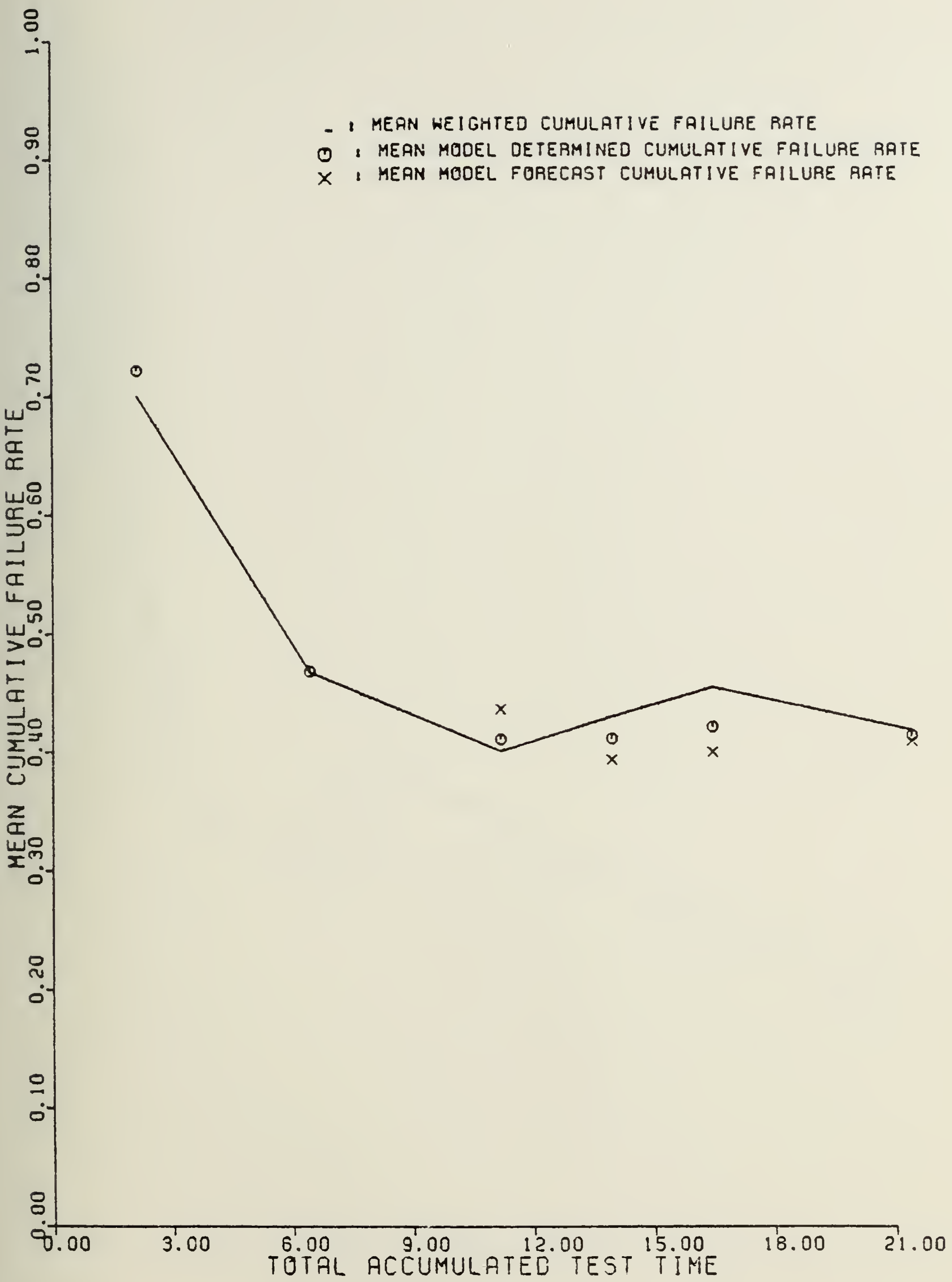

FIGURE 3.40

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD13: 6 PHASES, 5 TESTS/PHASE

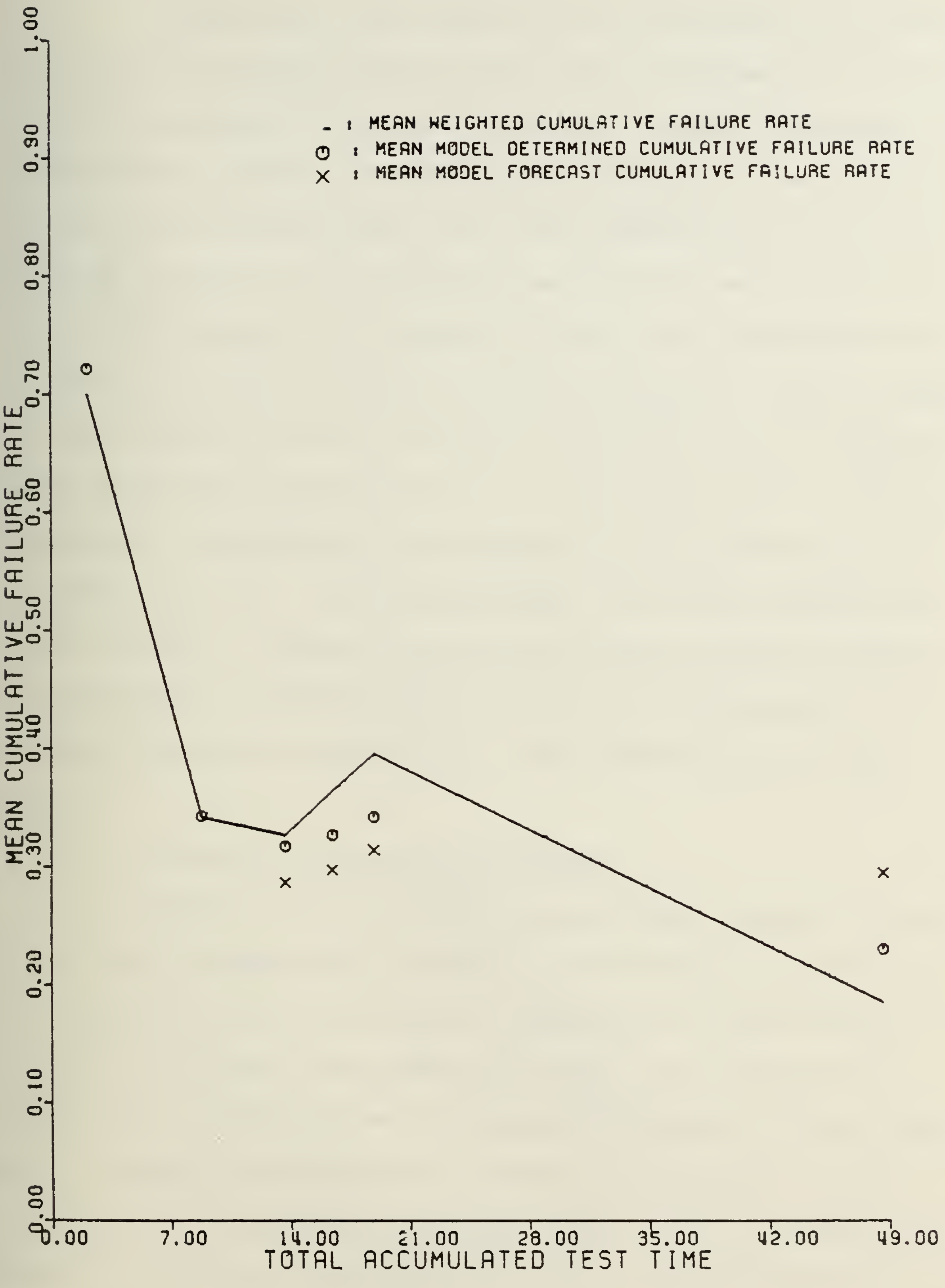

FIGURE 3.41

CUMULATIVE RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD14: 6 PHASES, 5 TESTS/PHASE

Figures 3.23, 3.24, 3.40, and 3.41 indicate that in instances where reliability growth progress patterns are interrupted by a period of reliability degradation the cumulative failure rate model lags behind the true underlying cumulative failure rate path with both its determined and forecast failure rate estimates. While this characteristic might be dismissed as the result of the "smoothing" nature of the model, the model's failure rate estimates are plotted with the mean test time weighted average cumulative failure rate which is itself a "smoothed" quantity. Hence, the model cannot be excused from failing to accurately portray the temporary reliability progress decay case. "Least data" cases ($NT_i = 5$) are presented and the performance was no worse for the ten and twenty test per phase simulations. The cumulative model failure rate estimates did display reliability progress patterns that resembled dampened versions of the true underlying cumulative failure rate paths, and the model's estimates were reasonably accurate for the final cumulative failure rate status.

b. Variability (Precision) Performance

Tables 3.3 thru 3.8 present the continuous cumulative failure rate reliability growth model's variability performance for determining the cumulative failure rate status of a component in a systems acquisition cycle. Entries in these tables ($\text{P.S.E.}_{\widehat{\lambda^*_{TT_i}}}$) were determined by equation 3.27. Tables 3.9 thru 3.14 present the cumulative failure rate model's variability performance for forecasting the cumulative failure rate status of a component one test phase ahead (i+1) of the current test phase (i). Entries in these tables ($\text{P.S.E.}_{\widehat{F\lambda_{TT_i}}}$) were determined by equation 3.35.

From figure 3.11 the mean model determined cumulative failure rate for the sixteenth phase of testing for lambda set 7 using five tests per phase was read as $\overline{\widehat{\lambda^*_{TT_{16}}}} = 0.45$. In table 3.3 under lambda set 7, phase 16 a value of 25% is entered for $P.S.E._{\widehat{\lambda^*_{TT_{16}}}}$. Therefore, the standard deviation of the mean model determined cumulative failure rate $\overline{\widehat{\lambda^*_{TT_{16}}}} = 0.45$ over one-hundred simulations (NSIMS = 100) was

$$S.D._{\widehat{\lambda^*_{TT_{16}}}} = \overline{\widehat{\lambda^*_{TT_{16}}}} \times \frac{P.S.E._{\widehat{\lambda^*_{TT_{16}}}}}{100} = 0.45 \times \frac{25}{100} = .1125 \qquad (3.36)$$

from equation 3.27.

As noted in Chapter II econometricians and cost estimators set goals of 10% to 20% for the percentage standard error measure of variability performance of their models. Since gross performance is being examined in this thesis, less stringent bench marks may be appropriate for evaluating the variability performance of the reliability growth models.

If a goal of 30% is set for the percentage standard error measure of variability performance, then a quick method of examining a model's variability performance is to determine in each variability performance table the first test phase at which the percentage standard error for all lambda sets (failure rate sets) meets the 30% goal. The earlier the test phase number at which this criterion is achieved the more confidence can be placed in the precision of the model. As with any sampling statistic, increasing sample size is expected to reduce variability. In the reliability testing procedure as the number of tests per phase NT_i is increased from five to ten to twenty the test phase number at which the 30% percentage standard error goal is achieved is

therefore expected to occur earlier in testing.

Tables 3.3 thru 3.14 list the variability performance of the continuous cumulative failure rate reliability growth model. In tables 3.3, 3.4, and 3.5 the cumulative failure rate model's variability performance for determined cumulative failure rate and sixteen phase lambda sets achieves the 30% goal at phase 10 for $NT_i = 5$, at phase 6 for $NT_i = 10$, and at phase 2 for $NT_i = 20$. The percentage standard error listed for phase 1 in any of the tables is for the failure rate estimates given by equation 3.24 vice for the cumulative model estimates which start with test phase 2. As can be seen the equation 3.24 estimates are subject to a great deal of variation. Tables 3.6, 3.7, and 3.8 show that in the case of the contracted six test phase cycles the cumulative model's variability performance for determined cumulative failure rate fails to achieve the 30% goal for $NT_i = 5$ and achieves the 30% goal at phase 5 for $NT_i = 10$ and phase 3 for $NT_i = 20$.

In tables 3.9, 3.10, and 3.11 the cumulative failure rate model's variability performance for forecast cumulative failure rate in sixteen test phase cycles achieves the 30% percentage standard error goal at phase 12 for $NT_i = 5$, at phase 7 for $NT_i = 10$, and at phase 5 for $NT_i = 20$. Of course forecasts are made for phases 3 and on only. Tables 3.12, 3.13, and 3.14 show that for the six phase lambda sets the cumulative model fails to achieve the 30% percentage standard error goal for variability performance in forecasting cumulative failure rates. The difficulties arise for lambda sets MOD7, MOD12, and MOD14. These are the lambda sets for which the mean test time weighted average underlying cumulative failure rate decreased most rapidly to approximately 0.20 for phase 6 in all cases, and these lambda sets are the ones for which the

cumulative model displayed the most difficulty in tracking the underlying cumulative failure rate progress path. Refer to figures 3.31, 3.39, and 3.41. Note in table 3.14 ($NT_i = 20$) that the 30% goal was achieved in phase 5 but lost in phase 6 for the three lambda sets enumerated above. If lambda sets MOD7, MOD12, and MOD14 were not considered, then the 30% percentage standard error goal was achieved in phase 6 for $NT_i = 10$.

B. CONTINUOUS INSTANTANEOUS FAILURE RATE RELIABILITY GROWTH MODEL

1. Model Description

The continuous instantaneous failure rate reliability growth model evaluated is of the form

$$\lambda = \frac{(1-a)b}{TT^a} \tag{3.37}$$

where

λ = instantaneous failure rate,

TT = total accumulated test time for all testing, and

a,b = model parameters that determine the shape of the model estimated instantaneous failure rate reliability progress path.

Total accumulated test time TT, the independent variable, is the same quantity utilized in the continuous cumulative failure rate reliability growth model, equation 3.1. Instead of the cumulative failure rate λ_{TT} modelled by equation 3.1, the instantaneous model of equation 3.37 models the unknown, underlying instantaneous failure rate λ_i. Therefore, rather than producing estimates of the underlying test time weighted average cumulative failure rate λ_{TT_i}, the instantaneous model was used to estimate the actual test phase instantaneous failure rates

TABLE 3.3

CUMULATIVE RELIABILITY GRCWTH MODEL VARIABILITY PERFORMANCE
16 PHASES, 5 TESTS/PHASE, DETERMINEC FAILURE RATE

							LAMBDA SET **********							
PHASE	1	2	3	4	5	6	7	8	9	10	11	12	13	14
*****	****	****	****	****	****	****	****	****	****	****	****	****	****	****
1	62	62	62	62	62	62	62	62	62	62	62	62	62	62
2	48	47	48	47	48	47	47	47	48	49	47	47	47	48
3	45	45	46	45	45	45	45	45	46	46	45	45	45	45
4	44	44	44	44	44	43	43	43	44	43	43	44	44	44
5	40	40	40	40	40	40	40	40	40	40	40	40	40	40
6	37	37	37	37	37	37	37	37	38	37	38	37	37	37
7	35	35	35	35	34	35	35	35	35	35	35	35	35	34
8	32	33	32	32	32	33	33	33	33	33	33	33	32	32
9	31	31	31	31	30	31	31	31	32	32	32	31	31	31
10	29	30	29	29	29	30	30	30	30	30	30	30	29	29
11	28	28	27	28	27	28	28	28	29	29	29	28	28	28
12	26	27	26	26	26	27	27	27	27	27	27	27	27	27
13	25	26	25	25	25	26	26	26	26	26	26	26	26	26
14	24	25	24	24	24	25	25	25	25	25	25	25	25	25
15	23	24	23	23	23	24	24	24	24	24	24	24	24	24
16	22	23	22	22	22	23	25	23	23	23	23	24	23	24

TABLE 3.4

CUMULATIVE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

16 PHASES, 10 TESTS/PHASE, DETERMINED FAILURE RATE

	LAMBDA SET													
PHASE	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	58	58	58	58	58	58	58	58	58	58	58	58	58	58
2	46	46	46	46	46	46	46	46	46	47	46	46	46	46
3	39	39	38	39	38	39	39	39	39	39	39	38	38	38
4	33	33	33	33	33	33	33	33	34	35	34	33	33	33
5	30	30	30	30	29	30	30	30	31	31	30	30	30	29
6	27	27	27	27	27	27	27	27	28	29	28	27	27	27
7	24	25	24	24	24	25	25	25	25	26	25	25	24	24
8	22	22	22	22	22	23	23	23	23	24	23	23	22	23
9	21	21	20	21	20	21	21	21	22	22	21	21	21	21
10	20	20	20	20	20	20	20	20	21	21	20	20	20	20
11	19	19	19	19	19	19	19	19	20	20	19	19	19	20
12	18	19	18	19	18	19	19	19	19	19	19	19	19	19
13	18	18	18	18	18	18	18	18	18	18	18	18	18	18
14	17	18	17	18	17	18	18	18	18	18	18	18	18	18
15	17	17	17	17	17	17	18	17	17	17	17	18	17	17
16	16	16	16	17	16	17	19	17	17	17	17	18	17	18

TABLE 3.5

CUMULATIVE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE
16 PHASES, 20 TESTS/PHASE, DETERMINED FAILURE RATE

	LAMBDA SET													
PHASE	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	42	42	42	42	42	42	42	42	42	42	42	42	42	42
2	27	27	27	27	27	27	27	27	27	27	27	27	27	27
3	23	23	23	23	23	23	23	23	23	23	23	23	23	23
4	21	21	21	21	21	21	21	21	21	21	21	21	21	21
5	20	20	20	20	20	20	20	20	20	19	20	20	20	20
6	19	19	19	19	19	19	19	19	19	18	19	19	19	19
7	18	18	18	18	18	18	18	18	18	17	18	18	17	17
8	17	17	17	17	17	17	17	17	17	16	17	17	16	16
9	15	16	15	16	15	16	16	16	16	15	16	16	15	15
10	15	15	15	15	15	15	15	15	15	15	15	15	15	15
11	14	14	14	14	14	15	15	15	15	14	15	14	14	14
12	14	14	14	14	14	14	14	14	14	14	14	14	14	14
13	13	14	13	14	13	14	14	14	14	14	14	14	13	13
14	13	13	13	13	13	13	14	13	13	13	14	13	13	13
15	13	13	13	13	12	13	13	13	13	13	13	14	13	13
16	12	12	12	12	12	13	14	13	13	13	13	14	12	14

TABLE 3.6

CUMULATIVE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

6 PHASES, 5 TESTS/PHASE, DETERMINED FAILURE RATE

	LAMBDA SET													
PHASE	MOD1	MOD2	MOD3	MOD4	MOD5	MOD6	MOD7	MOD8	MOD9	MOD10	MOD11	MOD12	MOD13	MOD14
1	65	65	65	65	65	65	65	65	65	65	65	65	65	65
2	47	46	47	46	47	46	46	46	46	48	46	46	46	47
3	42	42	42	42	42	41	41	41	42	42	42	42	42	42
4	40	40	40	40	40	40	40	39	40	40	40	40	40	40
5	37	37	37	37	37	37	37	37	37	37	37	37	37	37
6	36	35	36	36	36	36	39	35	35	35	36	39	36	39

TABLE 3.7

CUMULATIVE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

6 PHASES, 10 TESTS/PHASE, DETERMINED FAILURE RATE

	LAMBDA SET													
PHASE	MOD1	MOD2	MOD3	MOD4	MOD5	MOD6	MOD7	MOD8	MOD9	MOD10	MOD11	MOD12	MOD13	MOD14
1	46	46	46	46	46	46	46	46	46	46	46	46	46	46
2	39	39	39	39	39	39	39	39	39	40	39	39	39	39
3	35	35	35	35	35	35	35	35	35	35	35	35	35	35
4	31	31	31	31	31	31	31	31	31	31	31	31	31	32
5	27	27	27	26	26	27	27	27	27	28	27	27	28	28
6	25	25	25	25	25	25	27	25	25	26	25	27	26	27

TABLE 3.8

CUMULATIVE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE
6 PHASES, 20 TESTS/PHASE, DETERMINED FAILURE RATE

							LAMBDA SET							
							**********							
PHASE	MOD1	MOD2	MOD3	MOD4	MOD5	MOD6	MOD7	MOD8	MOD9	MOD10	MOD11	MOD12	MOD13	MOD14
*****	****	****	****	****	****	****	****	****	****	*****	*****	*****	*****	*****
1	37	37	37	37	37	29	37	37	37	37	37	37	37	37
2	30	29	30	30	30	37	29	29	30	30	29	30	30	30
3	26	25	26	25	26	25	25	25	26	26	25	25	26	26
4	22	23	22	22	22	22	22	22	23	23	23	23	23	23
5	21	21	21	21	21	21	21	21	21	21	21	21	21	21
6	19	19	18	18	18	19	18	19	19	19	19	19	19	20

TABLE 3.9

CUMULATIVE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE
16 PHASES, 5 TESTS/PHASE, FORECAST FAILURE RATE

	LAMBDA SET													
	**********													
PHASE	1	2	3	4	5	6	7	8	9	10	11	12	13	14
*****	****	****	****	****	****	****	****	****	****	****	****	****	****	****
3	81	79	80	83	85	79	78	78	65	52	74	85	88	87
4	55	54	54	56	56	55	54	54	51	46	53	54	55	55
5	51	50	51	52	53	50	50	50	48	44	49	50	51	51
6	45	45	45	47	47	45	45	45	43	40	45	44	45	44
7	40	41	40	41	41	41	41	41	40	38	40	40	40	39
8	37	37	37	38	37	38	38	38	37	36	37	37	36	35
9	34	35	34	35	34	35	35	35	35	34	35	34	33	33
10	32	33	32	33	32	33	33	33	33	32	33	32	31	31
11	30	31	30	31	30	31	32	31	31	31	32	31	30	29
12	29	29	28	29	28	30	30	29	29	29	30	29	28	28
13	27	28	27	28	27	28	29	28	28	28	29	28	27	27
14	26	26	26	27	26	27	28	27	27	27	27	27	26	26
15	25	25	25	26	24	26	27	26	26	25	26	27	25	25
16	24	24	24	24	23	25	30	25	24	24	25	29	24	27

TABLE 3.10

CUMULATIVE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

16 PHASES, 10 TESTS/PHASE, FORECAST FAILURE RATE

	LAMBDA SET **********													
PHASE *****	1 ****	2 ****	3 ****	4 ****	5 ****	6 ****	7 ****	8 ****	9 ****	10 ****	11 ****	12 ****	13 ****	14 ****
3	62	61	61	63	64	60	60	60	54	47	58	64	65	65
4	43	43	43	44	44	43	43	43	41	39	42	42	43	43
5	35	35	34	35	35	35	35	35	35	34	35	34	35	34
6	31	31	31	31	31	31	31	31	31	31	31	31	31	30
7	28	28	28	28	28	28	28	28	28	28	28	28	28	27
8	25	25	25	25	25	25	26	26	25	26	25	25	25	24
9	23	23	22	23	23	23	23	23	23	24	23	23	23	23
10	21	22	21	22	21	22	22	22	22	22	22	22	21	21
11	20	21	20	21	20	21	21	21	21	21	21	21	20	20
12	20	20	19	20	20	20	20	20	20	20	20	20	19	20
13	19	19	19	20	19	20	20	19	19	19	20	19	19	19
14	19	19	18	20	18	19	20	19	18	18	19	19	18	18
15	18	18	18	19	18	19	20	18	18	18	19	20	18	18
16	17	18	17	18	17	18	24	18	17	17	18	22	17	20

TABLE 3.11

CUMULATIVE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

16 PHASES, 20 TESTS/PHASE, FORECAST FAILURE RATE

	LAMBDA SET **********													
PHASE *****	1 ****	2 ****	3 ****	4 ****	5 ****	6 ****	7 ****	8 ****	9 ****	10 ****	11 ****	12 ****	13 ****	14 ****
3	59	56	58	61	65	55	55	53	40	28	49	64	68	67
4	32	31	32	33	34	31	31	31	27	23	30	31	32	33
5	27	26	26	27	28	26	26	26	24	21	25	25	26	26
6	24	24	24	25	25	24	24	23	22	20	23	23	23	23
7	21	21	21	22	22	21	21	21	20	19	21	21	21	20
8	19	19	19	20	20	20	20	19	19	18	19	19	19	18
9	18	18	18	19	18	18	18	18	18	17	18	18	17	17
10	17	17	16	17	17	17	17	17	16	16	17	16	16	16
11	16	16	16	16	16	16	16	16	16	15	16	16	15	15
12	15	15	15	16	15	16	16	15	15	15	16	15	14	14
13	15	15	14	15	14	15	15	15	15	14	15	15	14	14
14	14	14	14	15	14	14	15	14	14	14	15	15	14	14
15	14	14	13	14	13	14	15	14	14	13	14	15	13	13
16	13	13	13	14	13	14	17	13	13	13	14	17	13	15

TABLE 3.12

CUMULATIVE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

6 PHASES, 5 TESTS/PHASE, FORECAST FAILURE RATE

							LAMBDA SET							
PHASE	MOD1	MOD2	MOD3	MOD4	MOD5	MOD6	MOD7	MOD8	MOD9	MOD10	MOD11	MOD12	MOD13	MOD14
3	71	74	67	85	79	78	77	74	62	49	69	66	67	55
4	53	53	51	61	55	56	58	53	49	43	51	50	46	44
5	45	45	44	48	44	47	50	45	43	40	49	44	42	41
6	40	41	39	43	39	43	69	41	39	37	42	63	41	57

TABLE 3.13

CUMULATIVE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE
6 PHASES, 10 TESTS/PHASE, FORECAST FAILURE RATE

							LAMBDA SET							
PHASE	MOD1	MOD2	MOD3	MOD4	MOD5	MOD6	MOD7	MOD8	MOD9	MOD10	MOD11	MOD12	MOD13	MOD14
3	46	47	45	50	49	48	48	47	43	39	46	45	45	41
4	38	39	37	41	39	40	41	39	37	35	38	38	36	35
5	33	34	33	35	33	35	36	34	33	31	35	33	32	32
6	28	29	28	30	27	30	52	29	28	27	29	45	29	39

TABLE 3.14

CUMULATIVE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

6 PHASES, 20 TESTS/PHASE, FORECAST FAILURE RATE

							LAMBDA SET							
PHASE	MOD1	MOD2	MOD3	MOD4	MOD5	MOD6	MOD7	MOD8	MOD9	MOD10	MOD11	MOD12	MOD13	MOD14
3	38	39	37	43	41	41	40	39	35	30	37	37	37	33
4	29	30	28	32	30	31	31	30	28	26	29	29	27	26
5	25	26	25	28	25	27	29	26	24	23	28	25	24	24
6	23	23	22	25	22	25	54	23	22	21	24	45	23	36

λ_i specified as inputs to the reliability testing procedure computer simulation. These are the failure rates of lambda sets contained in tables 3.1 and 3.2.

As can be seen in equation 3.37 and similar to the cumulative failure rate model, as total accumulated test time increases instantaneous failure rate is modelled as decreasing. Thus, reliability of components is portrayed as increasing in general. Assumptions regarding the signs of the values for the model parameters a and b in the instantaneous model are not as obvious as the assumptions might be for the signs of the model parameters α and β in the cumulative failure rate model.

A mathematical analysis of the instantaneous failure rate model is given in reference 2. A Monte Carlo simulation evaluation and comparison of the instantaneous failure rate model is contained in reference 5.

2. Reliability Testing Procedure

The reliability testing procedure of a system acquisition cycle appropriate to the continuous instantaneous failure rate reliability growth model is identical to the procedure described for the cumulative failure rate model in chapter III.A.2. Therefore, the test data necessary for evaluation of the instantaneous model were collected simultaneously with the collection of data for the evaluation of the cumulative model in a reliability testing procedure as graphically depicted in figure 3.1. No additional test results are required for evaluation of the instantaneous failure rate model.

3. Reliability Testing Procedure Computer Simulation

Since the reliability testing procedure applicable to the continuous and instantaneous failure rate models are identical, data for

evaluating both models were collected from a single computer simulation program as described in chapter III.A.3. Features of the reliability testing procedure computer simulation such as phase planned test time PTT_i determination and assignment of one-half a failure in those instances when no failures occurred during a test phase were in affect for the evaluation of both the failure rate models. Also, the same twenty-eight lambda sets (failure rate sets) given in tables 3.1 and 3.2 and the same number of simulation replications (NSIMS = 100) were utilized for both model evaluations.

4. Computer Simulation Data Manipulation

The continuous instantaneous failure rate reliability growth model of equation 3.37 may be written as

$$\lambda_i = (1-a_i)b_i(TT_i)^{-a_i} . \qquad (3.38)$$

As noted in equation 3.16 an estimate of the unknown, underlying instantaneous failure rate of a component for any given test phase of a reliability testing procedure corrected for planned test time bias is given by

$$\widehat{\lambda}_i = \frac{2NT_i}{1 + 2NT_i} \cdot \frac{F_i}{T_i} \qquad (3.39)$$

for i = 1, 2, 3, ... , K where T_i is the total test time accumulated for the number of components NT_i tested during the phase and F_i is the number of component failures occurring during the phase.

If the logarithmic transformation is applied to equation 3.38, then

$$\ln \lambda_i = \ln (1-a_i)b_i - a_i \ln TT_i . \qquad (3.40)$$

To obtain the ordinary least squares regression estimates for a_i and b_i the data pairs $(\ln \widehat{\lambda}_i, \ln TT_i)$ are employed from the results of the reliability testing procedure computer simulation. Let

$$Y_i = \ln \widehat{\lambda}_i ,$$

$$X_i = \ln TT_i ,$$

$$\bar{Y}_i = \frac{1}{i} \sum_{j=1}^{i} Y_j , \text{ and}$$

$$\bar{X}_i = \frac{1}{i} \sum_{j=1}^{i} X_j$$

for i = 1, 2, 3, ... , K. Then equation 3.40 becomes

$$Y_i = \ln b_i(1-a_i) - a_i X_i \tag{3.41}$$

for i = 1, 2, 3, ... , K. By the derivation presented in Appendix A the ordinary least squares regression estimates for the instantaneous failure rate model parameters a_i and b_i are

$$\widehat{a}_i = \frac{\sum_{j=1}^{i} X_j Y_j - \bar{Y}_i \sum_{j=1}^{i} X_j}{\bar{X}_i \sum_{j=1}^{i} X_j - \sum_{j=1}^{i} X_j^2} \quad \text{and} \tag{3.42}$$

$$\widehat{b}_i = \frac{1}{1-\widehat{a}_i} \exp\,(\bar{Y}_i + \widehat{a}_i \bar{X}_i) \tag{3.43}$$

for i = 2, 3, 4, ... , K. Note that as with the cumulative failure rate model the regression requires at a minimum observations on the results

of two test phases; thus, model parameter estimates are made for the second thru the K^{th} test phase. The instantaneous failure rate estimate given by equation 3.39 was used for the first phase of testing; i.e.,

$$\widehat{\lambda}_{T_1} = \widehat{\lambda}_1 = \frac{2NT_1}{1 + 2NT_1} \cdot \frac{F_1}{T_1} . \qquad (3.44)$$

As the instantaneous failure rate model parameter estimates were obtained, they were applied to the model in equation 3.38 to obtain model determined estimates for the instantaneous failure rate reliability status at the end of each phase of testing; i.e.,

$$\widehat{\lambda}_{T_i} = (1-\widehat{a}_i)\widehat{b}_i(TT_i)^{-\widehat{a}_i} \qquad (3.45)$$

for i = 2, 3, 4, ... , K. Again, the simulation index r is left implicit (r = 1, 2, 3, ... , NSIMS = 100).

Upon completion of all computer simulations (NSIMS = 100) for each specified lambda set, performance statistics were computed as

$$\overline{\widehat{\lambda}}_{T_i} = \frac{1}{NSIMS} \sum_{r=1}^{NSIMS} \widehat{\lambda}_{T_{i,r}} , \qquad (3.46)$$

$$S.D._{\widehat{\lambda}_{T_i}} = \sqrt{\frac{1}{NSIMS-1} \sum_{r=1}^{NSIMS} (\widehat{\lambda}_{T_{i,r}} - \overline{\widehat{\lambda}}_{T_i})^2} , \text{ and} \qquad (3.47)$$

$$P.S.E._{\widehat{\lambda}_{T_i}} = \frac{S.D._{\widehat{\lambda}_{T_i}}}{\overline{\widehat{\lambda}}_{T_i}} \times 100 \qquad (3.48)$$

for i = 1, 2, 3, ... , K. Also, in order to evaluate the forecasting performance of the instantaneous failure rate model the same method of

computing an estimate of the total accumulated test time for a one test phase in the future failure rate estimate utilized for the cumulative failure rate model in equation 3.31 was utilized for the instantaneous failure rate model forecasts. Hence,

$$\widehat{F\lambda}_{T_{i+1}} = (1-\hat{a}_i)\hat{b}_i(\widehat{TT}_{i+1})^{-\hat{a}_i} \quad \text{where} \tag{3.49}$$

$$\widehat{TT}_{i+1} = TT_i + (NT_{i+1} \times PTT_{i+1}) \tag{3.50}$$

for i = 2, 3, 4, ... , K. Note again that forecasts were made for test phases 3 thru K as was the case with the cumulative failure rate model. Forecast statistics were then computed as

$$\overline{\widehat{F\lambda}}_{T_i} = \frac{1}{NSIMS} \sum_{r=1}^{NSIMS} \widehat{F\lambda}_{T_{i,r}}, \tag{3.51}$$

$$S.D._{\widehat{F\lambda}_{T_i}} = \sqrt{\frac{1}{NSIMS-1} \sum_{r=1}^{NSIMS} (\widehat{F\lambda}_{T_{i,r}} - \overline{\widehat{F\lambda}}_{T_i})^2}, \text{ and} \tag{3.52}$$

$$P.S.E._{\widehat{F\lambda}_{T_i}} = \frac{S.D._{\widehat{F\lambda}_{T_i}}}{\overline{\widehat{F\lambda}}_{T_i}} \times 100 \tag{3.53}$$

for i = 3, 4, 5, ... , K.

5. Model Performance

a. Accuracy Performance

Figures 3.42 thru 3.77 depict the accuracy performance of the continuous instantaneous failure rate reliability growth model for selected tests per phase NT_i cases of the lambda sets listed in tables 3.1 and 3.2. In each graph the specified underlying instantaneous

failure rate λ_i from table 3.1 or 3.2 (——, solid line), the mean model determined instantaneous failure rate $\overline{\widehat{\lambda}}_{T_i}$ from equation 3.46 (⊙ , circles), and the mean model forecast instantaneous failure rate $\overline{\widehat{F\lambda}}_{T_i}$ from equation 3.51 (X , crosses) are all plotted against the mean total accumulated test time $\overline{TT}_i$ from equation 3.29 for each phase of the specified reliability testing procedure of the acquisition cycle. While the specified underlying instantaneous failure rate is plotted as a smooth, continuous line for purposes of contrast, it should be noted from both figures 3.1 and 3.2 and the testing procedure design that the specified underlying instantaneous failure rate is actually a step function in shape with discontinuities occurring at the end of each test phase.

Examining figure 3.47 for example at a mean total accumulated test time of approximately 205.0 test time units the specified instantaneous failure rate is 0.30, the mean model determined instantaneous failure rate is approximately 0.24, and the mean model forecast instantaneous failure rate is approximately 0.19. As noted on the graph the accuracy performance plotted is for the lambda set 6, 16 test phase, 20 tests per phase reliability testing procedure simulation. Since the point examined is for the last test phase ($i = K = 16$), table 3.1 shows that for lambda set 6, test phase 16 the specified instantaneous failure rate is 0.30 as expected.

The instantaneous failure rate model frequently yielded mean forecast failure rates that were "off-the-scale" of the accuracy performance graphs; i.e., mean forecast failure rates greater than 1.0. These off-the-scale mean estimates ranged from failure rates only slightly greater than 1.0 to mean estimated failure rates of magnitudes in excess

of 10^6. The extreme values occurred only for the first forecast and occasionally for the second forecast (test phases 3 and 4). Regardless of the magnitude of these off-the-scale forecast failure rates, all points of magnitude greater than 1.0 were plotted at 1.0.

Figures 3.42 thru 3.46 and 3.60 thru 3.64 display the instantaneous failure rate model's performance for the "nice" underlying failure rate progress paths. The model, while discerning the shape of these "nice" underlying failure rate patterns, is consistently optimistic in both determining the current failure rate status and forecasting the next test phase failure rate, especially in the case of the longer sixteen phase reliability testing procedures. The model performs more accurately for the more volatile six test phase procedures (figures 3.60 thru 3.64) than for the protracted sixteen test phase procedures (figures 3.42 thru 3.46). In fact mean determined failure rate performance is excellent for the shorter six test phase cases. Forecasting accuracy typically is off-the-scale for the first test phase improving rapidly by the third or fourth test phase. Once stabilized, the mean forecast failure rate generally is biased optimistically as expected from the method utilized to produce forecasts (equations 3.49 and 3.50).

In figures 3.47, 3.48, and 3.65 thru 3.67 accuracy of the instantaneous failure rate model is displayed for essentially linear underlying failure rate progress paths. Again, mean determined failure rate performance shows that the model detects the linear shape of the progress path with reasonable accuracy but provides consistently optimistic estimates of the underlying instantaneous failure rate. Forecasting accuracy, while consistent in the sixteen test phase procedures, is very poor being only around 50%-60% of the magnitude of the underlying

instantaneous failure rate for the majority of the test phases until the final one or two phases. For the six phase procedures forecasting performance is extremely erratic as can be seen in figures 3.65 thru 3.67. In fact forecasting performance actually degrades as more test information becomes available in the case of lambda set MOD6; i.e., as the number of tests per phase increases from five to twenty ($NT_i = 5$ vs. $NT_i = 20$, figure 3.65 vs. 3.66). Also, figure 3.67 is for a twenty tests per phase ($NT_i = 20$) procedure; and therefore, in keeping with the convention established for presenting accuracy performance graphs this figure shows the best performance of the instantaneous failure rate model for lambda set MOD7. The five and ten tests per phase performance for lambda set MOD7 were worse than the performance shown in figure 3.67.

Figures 3.49 thru 3.52 and 3.69 thru 3.17 portray the instantaneous failure rate model's accuracy performance for permanently stagnated reliability status cases. The most vivid contrast for these cases is between situations in which failure rate stagnates at a high value ($\lambda_i = 0.70$; figures 3.49, 3.68, and 3.69) versus stagnation at a low rate ($\lambda_i = 0.05$; figures 3.52 and 3.71). When the underlying failure rate stagnates at a high value, the instantaneous model's mean determined failure rate estimates are very erratic ($\lambda_i = 0.70$; figures 3.49 and 3.68) and the mean forecast estimates are virtually useless from a magnitude consideration being, even once stabilized, approximately 40%-50% of the magnitude of the true underlying instantaneous failure rates. Note that figure 3.49 is for a "best case" situation of $NT_i = 20$. Forecasts at best give an indication of stagnation when the stagnation occurs at a high true underlying value. In the case of lambda set MOD8 determined estimated failure rate improves significantly when more testing

is performed; i.e., figure 3.69 versus figure 3.68 ($NT_i = 20$ vs. $NT_i = 5$). On the other hand, when the true underlying failure rate stagnates at a low point, both the model's determined and forecast accuracy performance appear to be excellent ($\lambda_i = 0.05$, figures 3.52 and 3.71, $NT_i = 5$). Figures 3.50, 3.51, and 3.70 show for an intermediate stagnation level ($\lambda_i = 0.30$) how determined and forecast failure rate performance of the instantaneous model transitions from poor to excellent.

The contrast between the underlying failure rate permanent stagnation cases points out a general performance characteristic of the instantaneous failure rate model. Because the model produces consistent optimistically biased determined and forecast failure rate estimates, as the true underlying failure rate progress path approaches low failure rates the model's accuracy performance improves significantly by necessity since the estimates become "sandwiched" between the low true underlying failure rate and 0.0. This "closing of the brackets" effect may also produce favorable variability performance for the instantaneous failure rate model.

Figures 3.53, 3.54, 3.72, and 3.73 present the accuracy performance of the instantaneous failure rate model in the case of true reliability status progress interrupted by a period of stagnation. Only "best case" ($NT_i = 20$) graphs are shown. The model's mean determined instantaneous failure rate performance is fair for these cases and continues to exhibit consistent optimistic bias. Mean forecast failure rates generally are indicative of the underlying failure rate path trend only and are significantly off the mark in forecasting instantaneous failure rate magnitude. The "kick-down" in the mean forecast of the failure rate for the first phase after the period of stagnation in figure

3.53 (test phase 11 at approximately 130.0 total accumulated test time units) is more pronounced in the five and ten tests per phase (NT_i = 5, 10) cases for lambda sets 11 and 12.

For situations of temporary reliability status degradation figures 3.55 thru 3.59 and 3.74 thru 3.77 reveal that in these "worst situation" cases the instantaneous failure model performs surprisingly well in discerning the shape of the true underlying failure rate path, especially for the twenty tests per phase procedures (NT_i = 20; figures 3.57, 3.59, 3.75, and 3.77). During the period of failure rate degradation, the instantaneous model tends to produce determined failure rate estimates that overstate the degree of degradation. This is especially true in the "least data" cases (NT_i = 5, figures 3.55 and 3.58) of the sixteen phase testing procedures. This characteristic is not as pronounced for the six test phase procedures (NT_i = 5, figures 3.74 and 3.76). The model's mean forecast instantaneous failure rate estimates remain optimistic throughout the periods of degradation, but reflect the shape of the true underlying failure rate progress path with very credible fidelity. The figure series 3.55 thru 3.57 indicates how mean determined failure rate estimating performance improves with additional testing (NT_i = 5, 10, and 20) while mean forecast failure rate estimating performance is very stable after the third forecast for all test per phase sizes. Again, note in figures 3.59 and 3.77 that when the true instantaneous failure rate has decreased to a small magnitude (test phase 16), the instantaneous model's determined and forecast failure rate accuracy displays significant improvement.

b. Variability (Precision) Performance

The variability performance of the continuous instantaneous failure rate model is presented in tables 3.15 thru 3.26. Entries in

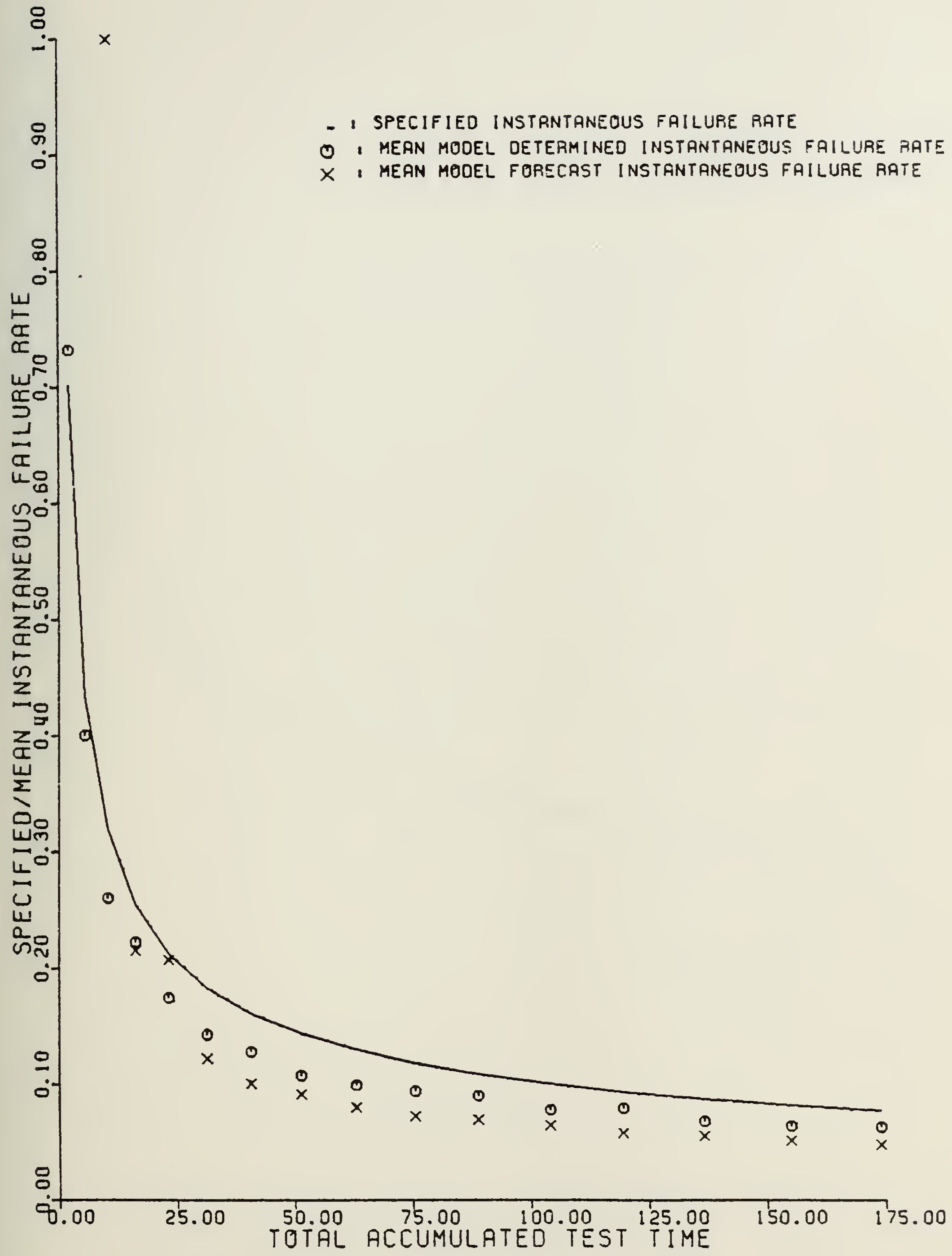

FIGURE 3.42

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 1: 16 PHASES, 5 TESTS/PHASE

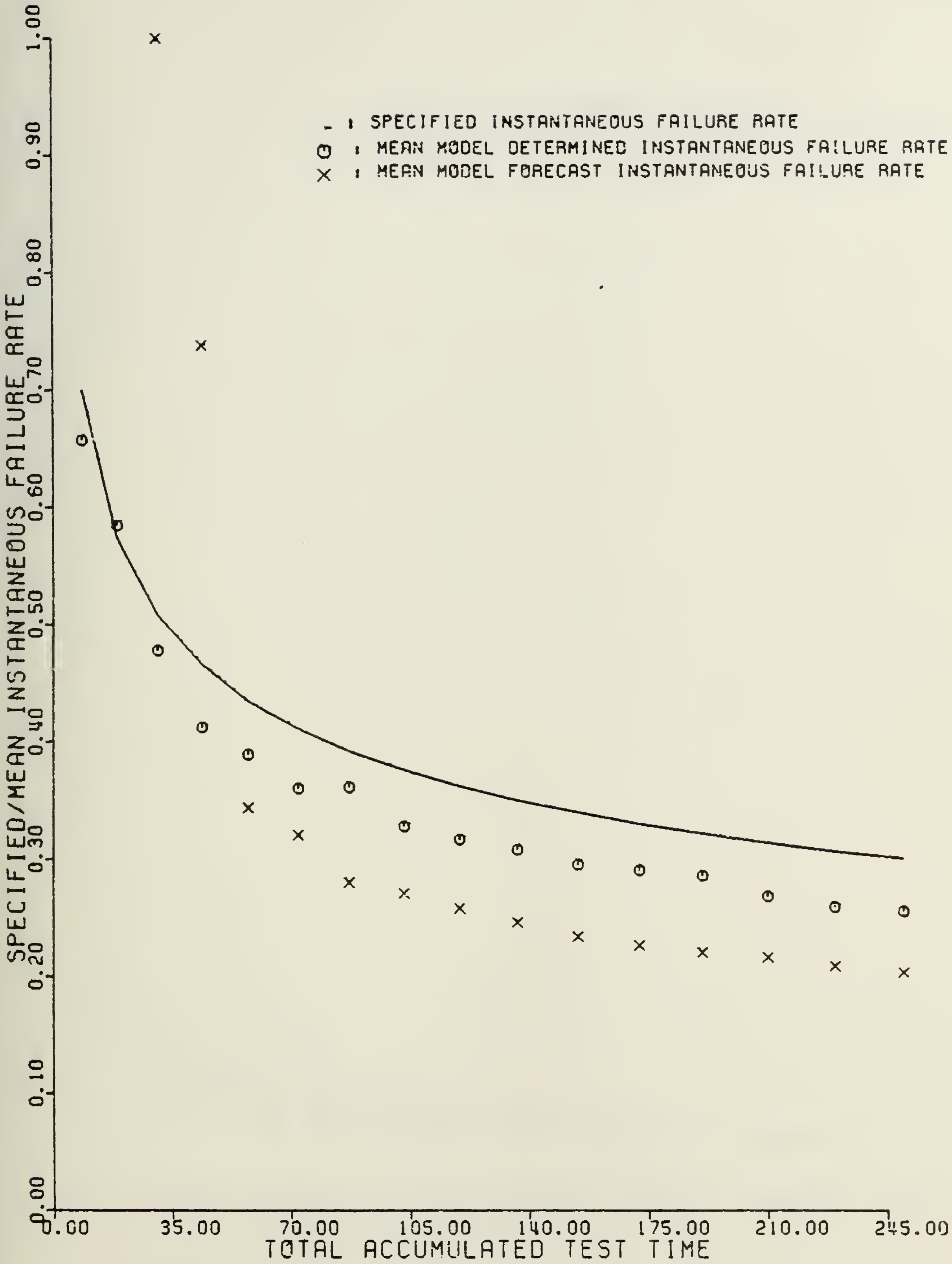

FIGURE 3.43

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 2: 16 PHASES, 20 TESTS/PHASE

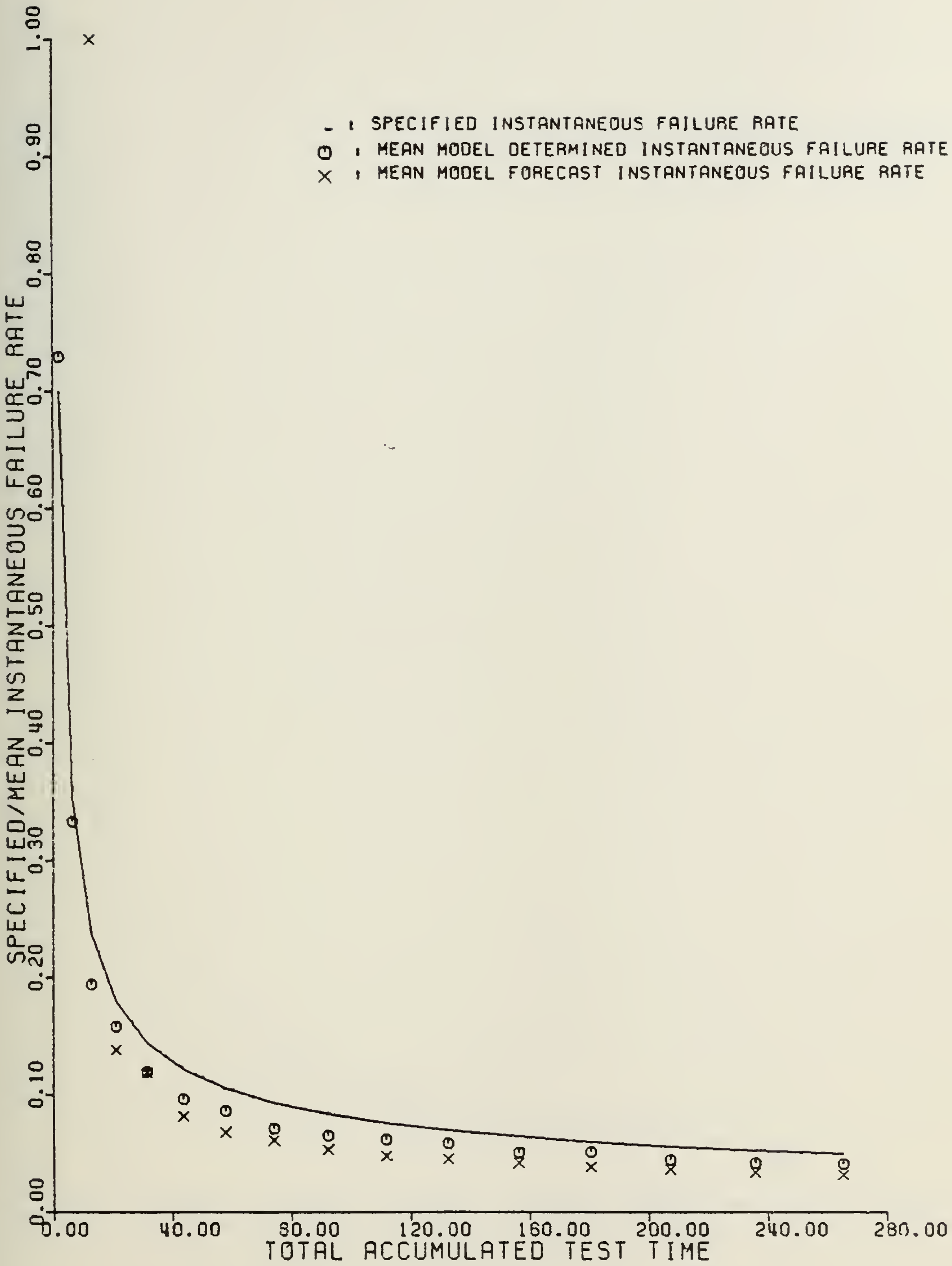

FIGURE 3.44

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 3: 16 PHASES, 5 TESTS/PHASE

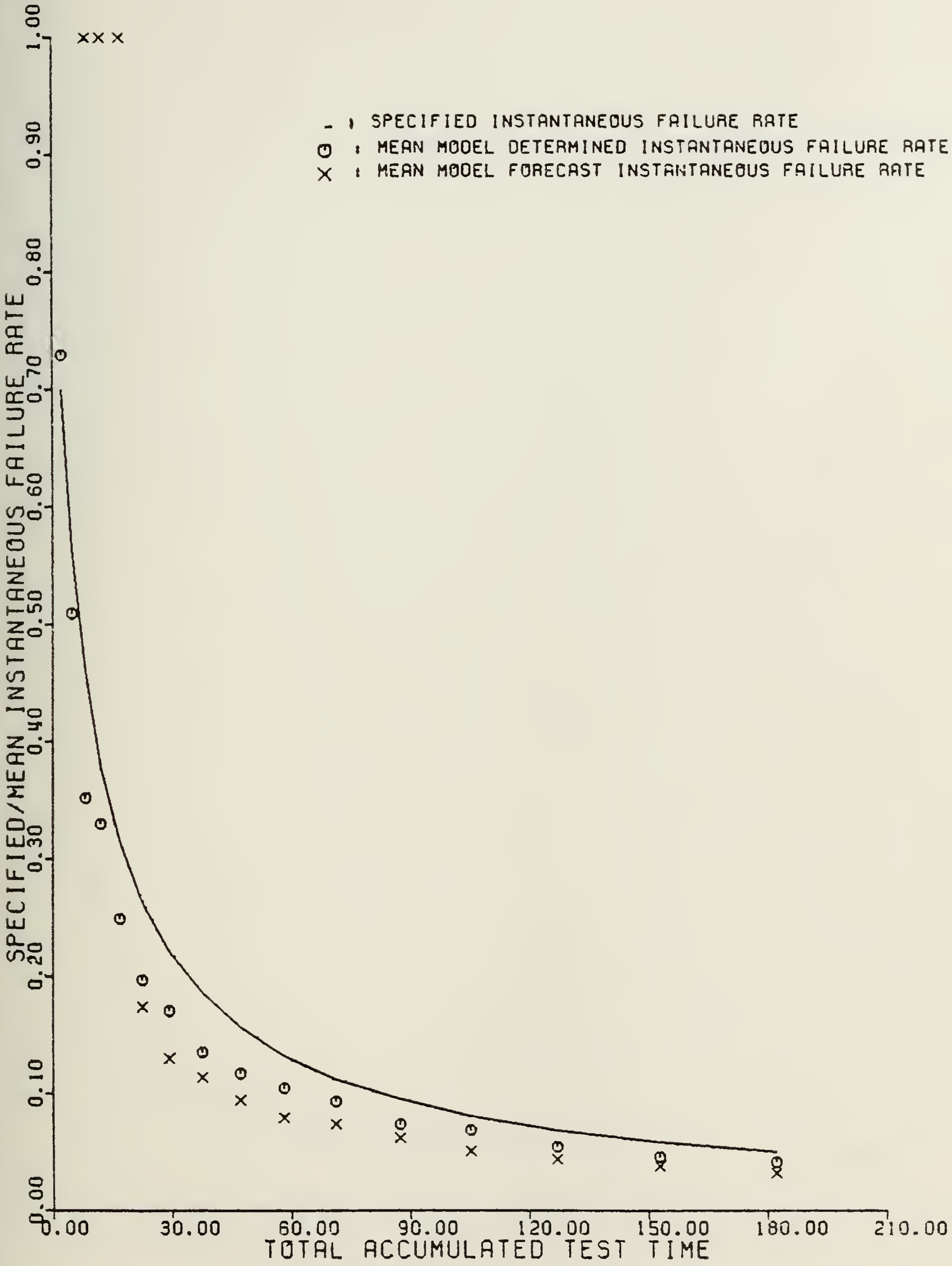

FIGURE 3.45

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 4: 16 PHASES, 5 TESTS/PHASE

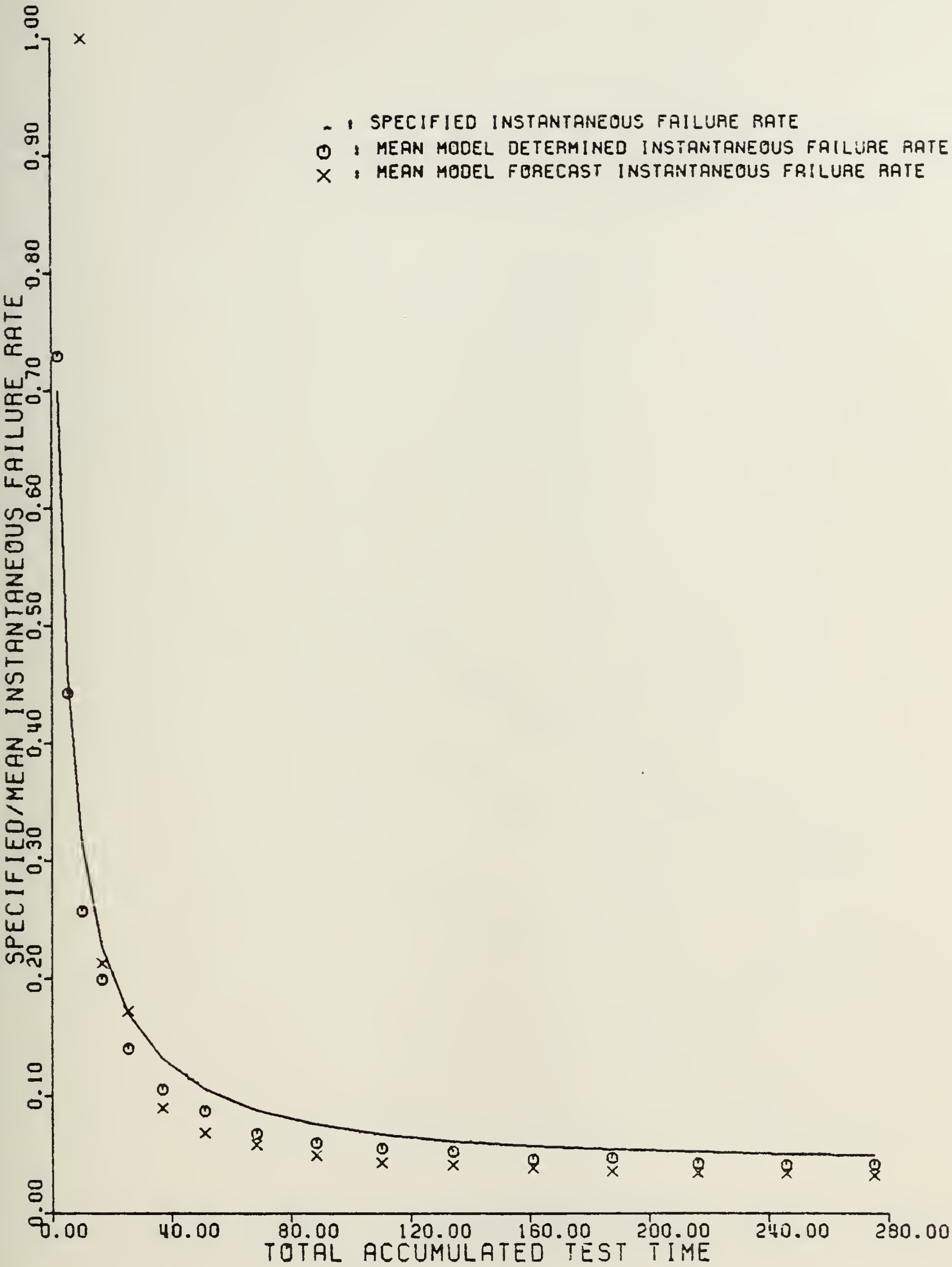

FIGURE 3.46

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 5: 16 PHASES, 5 TESTS/PHASE

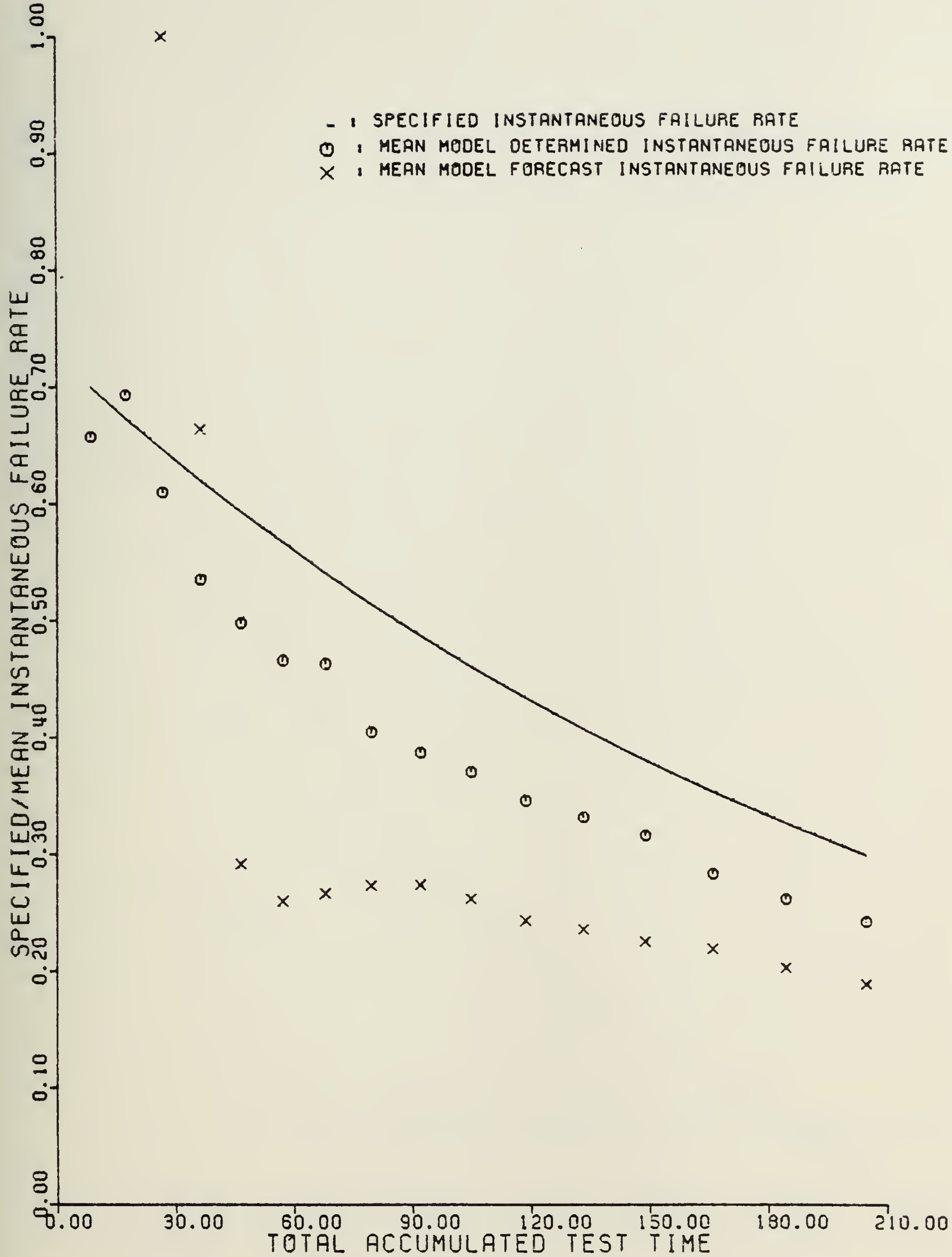

FIGURE 3.47

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 6: 16 PHASES, 20 TESTS/PHASE

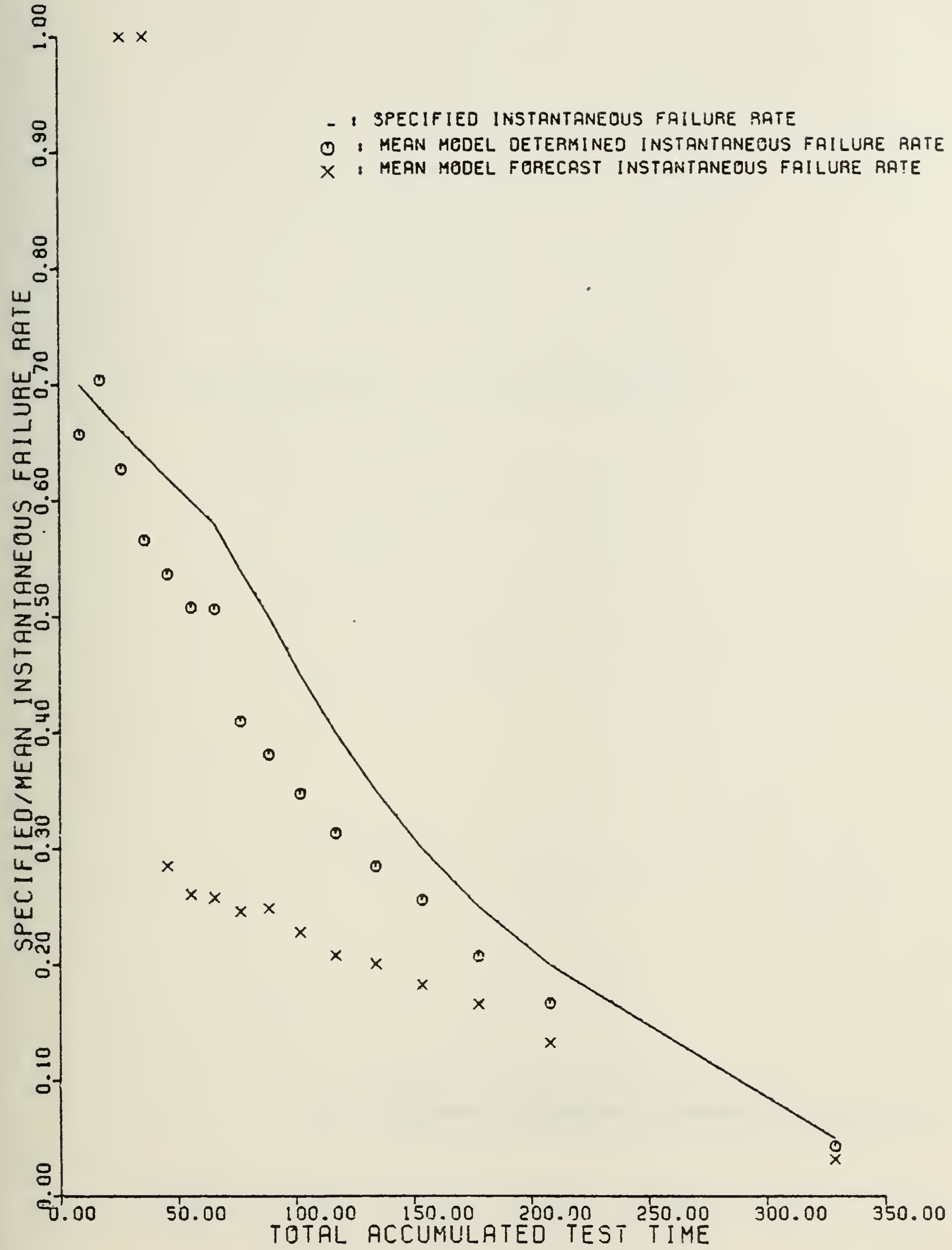

FIGURE 3.48

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 7: 16 PHASES, 10 TESTS/PHASE

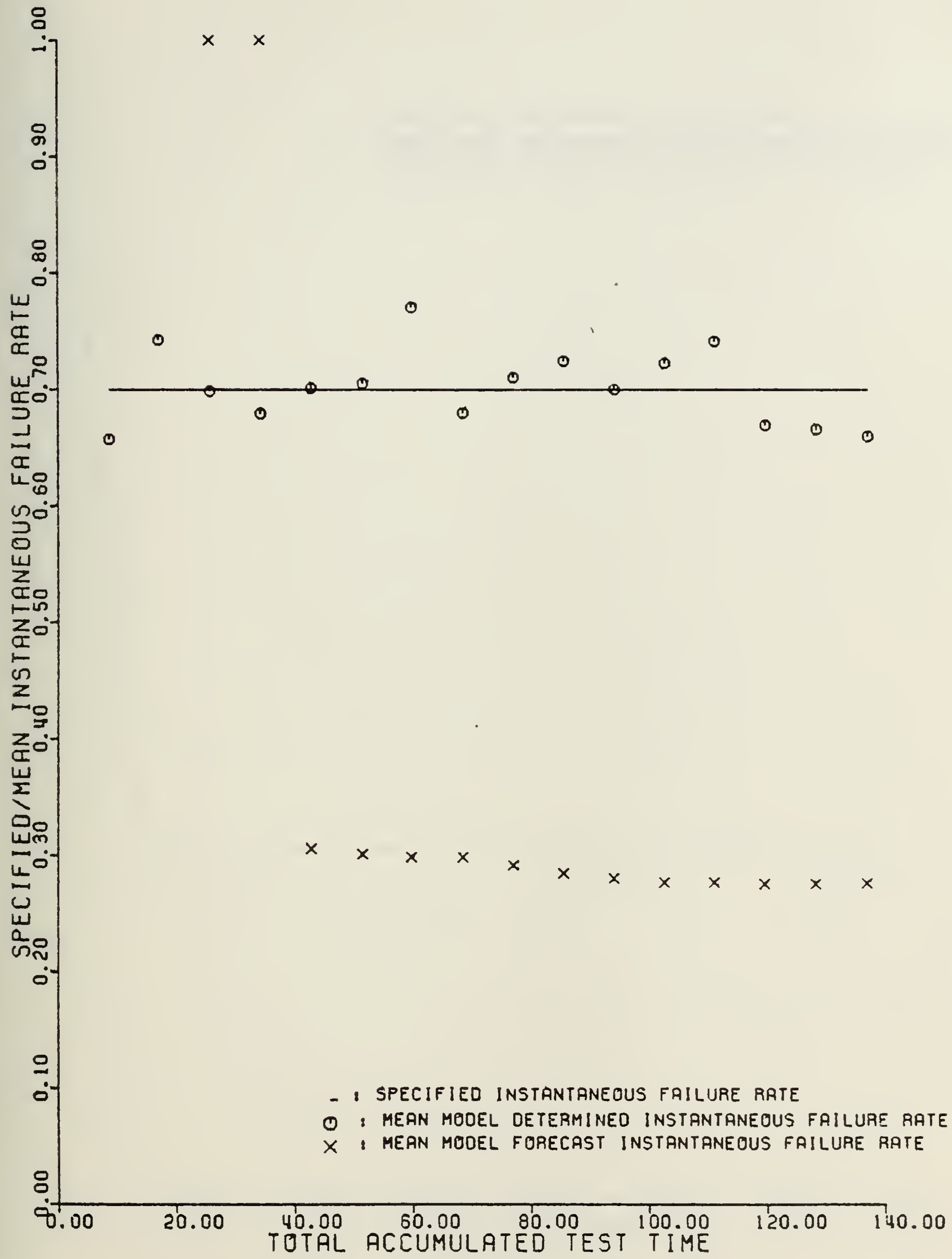

FIGURE 3.49

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 8: 16 PHASES, 20 TESTS/PHASE

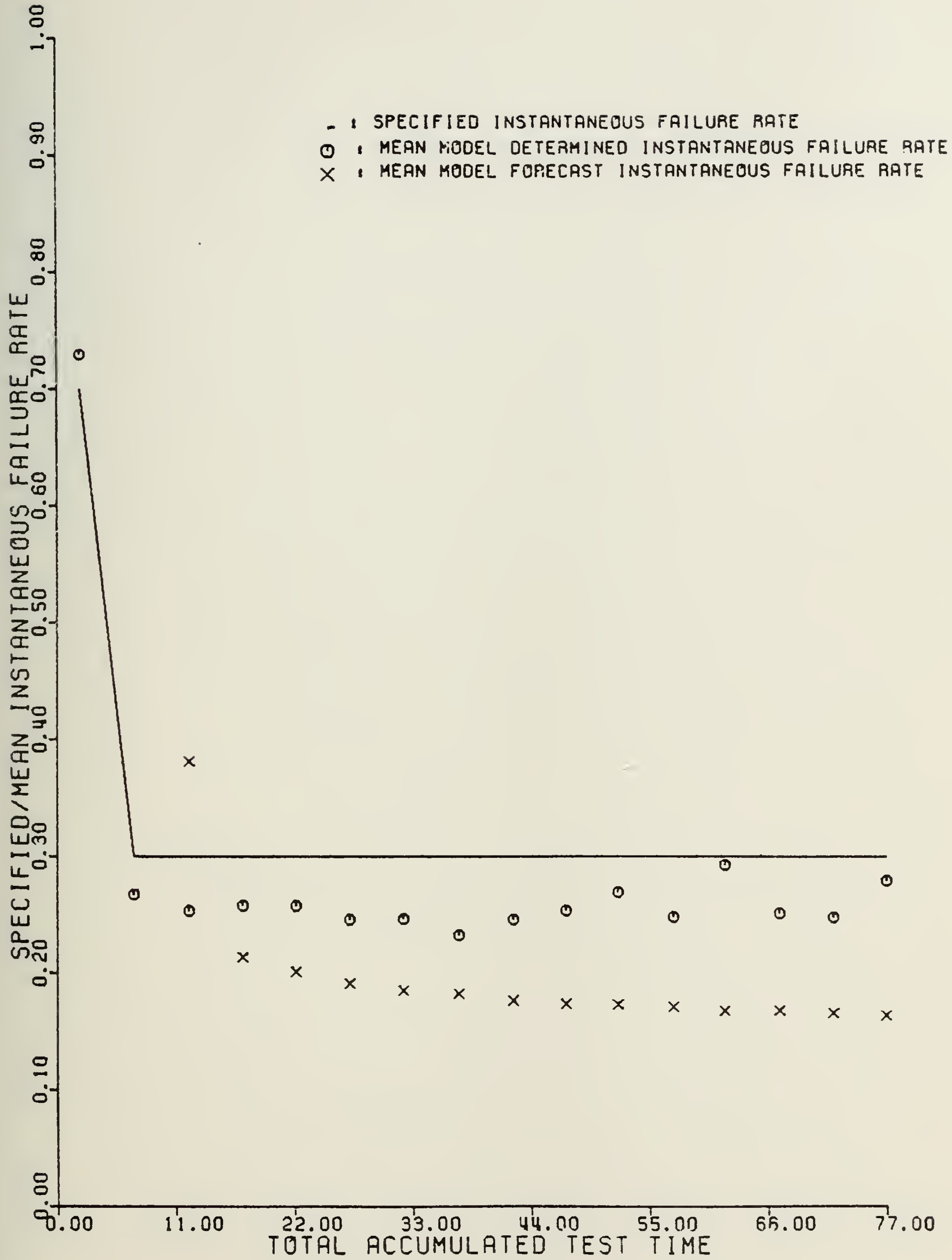

FIGURE 3.50

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 9: 16 PHASES, 5 TESTS/PHASE

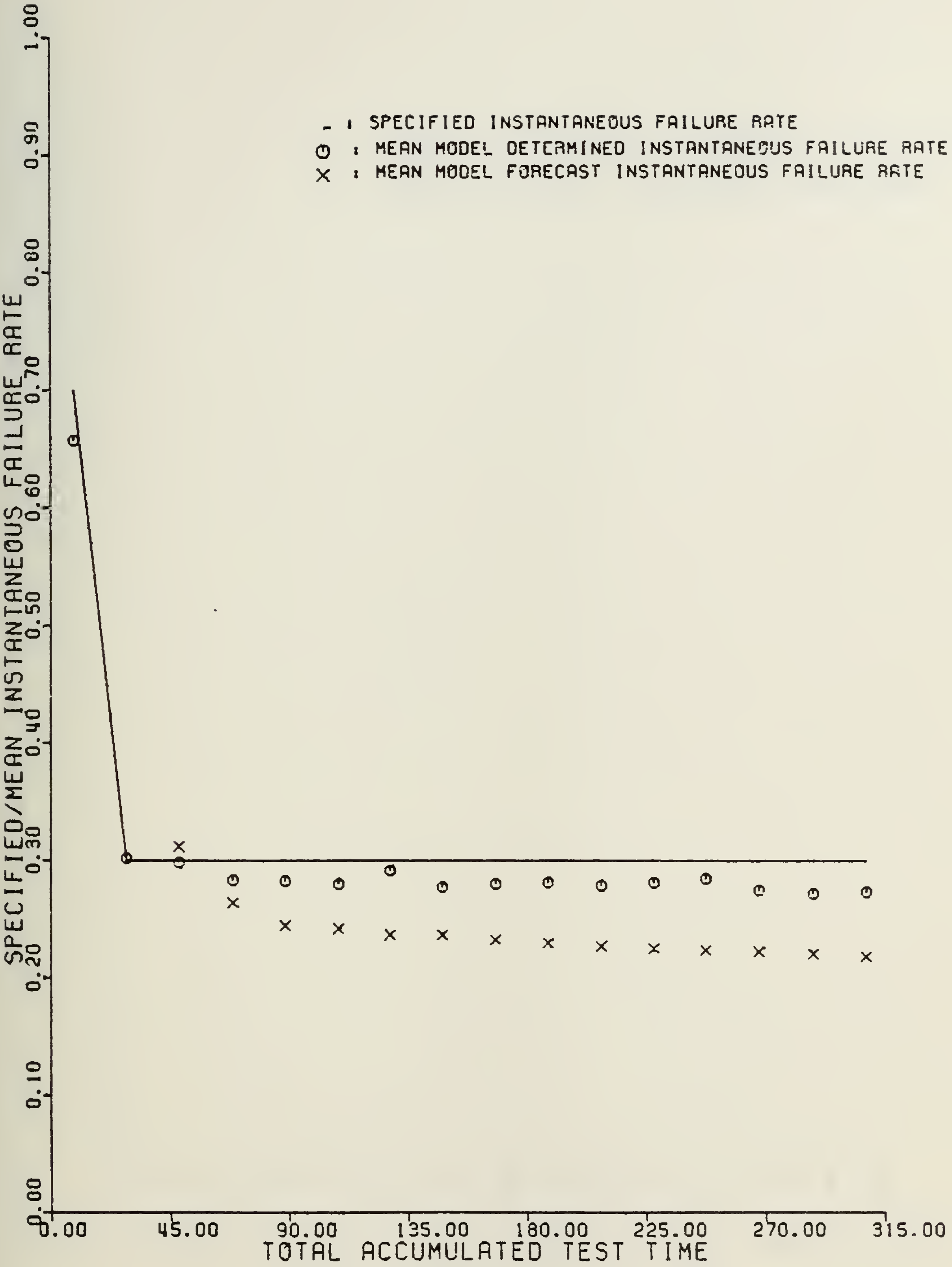

FIGURE 3.51

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 9: 16 PHASES, 20 TESTS/PHASE

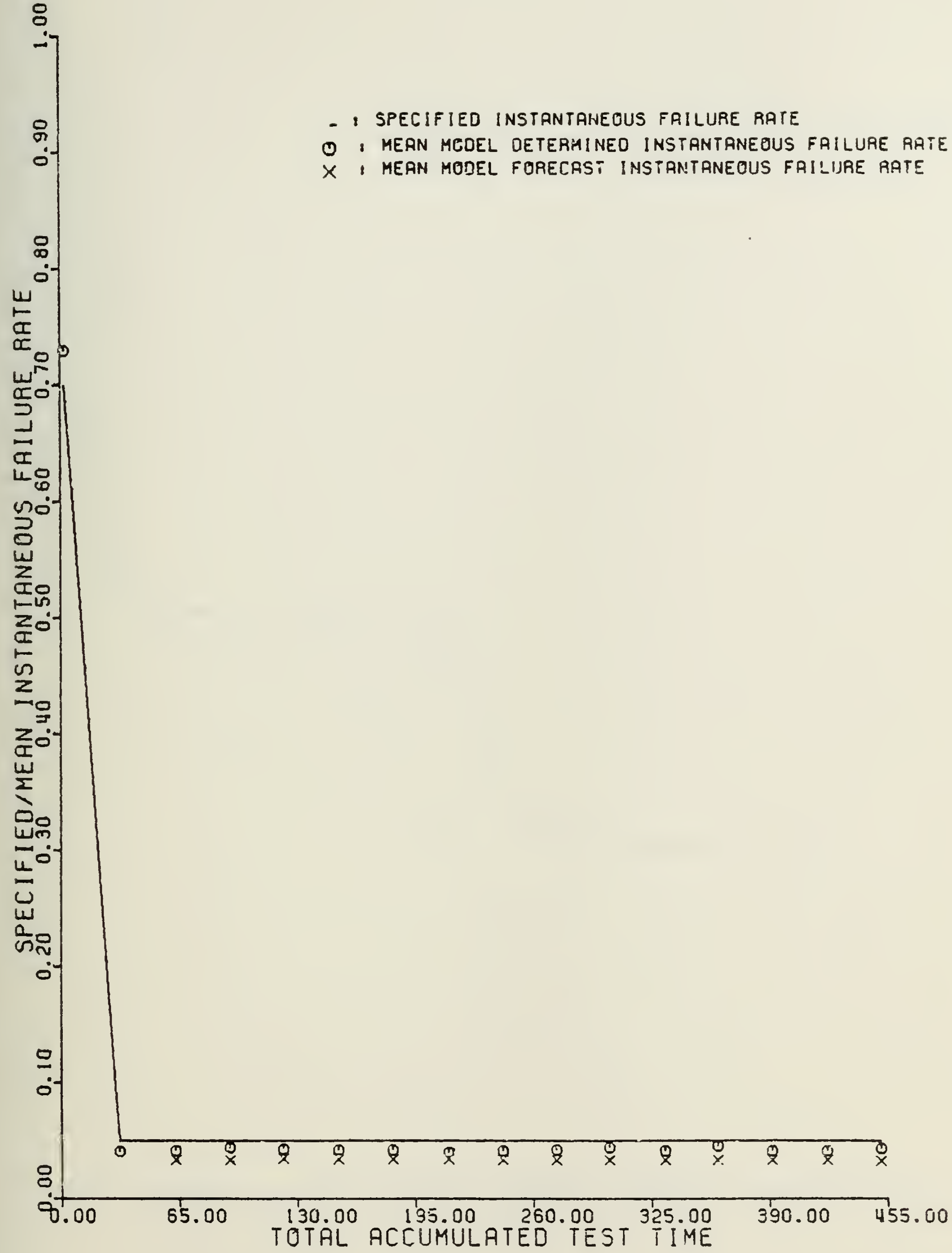

FIGURE 3.52

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 10: 16 PHASES, 5 TESTS/PHASE

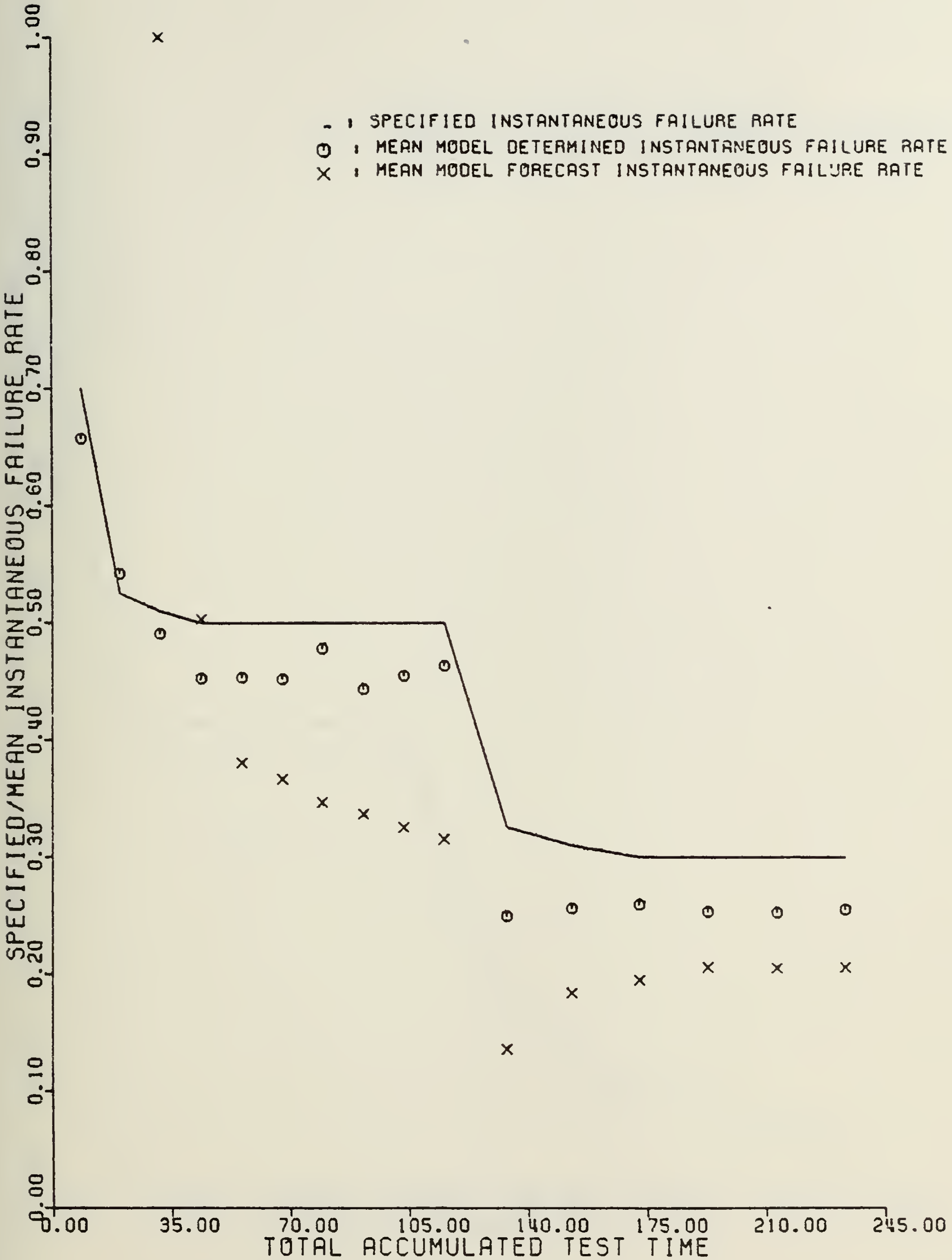

FIGURE 3.53

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 11: 16 PHASES, 20 TESTS/PHASE

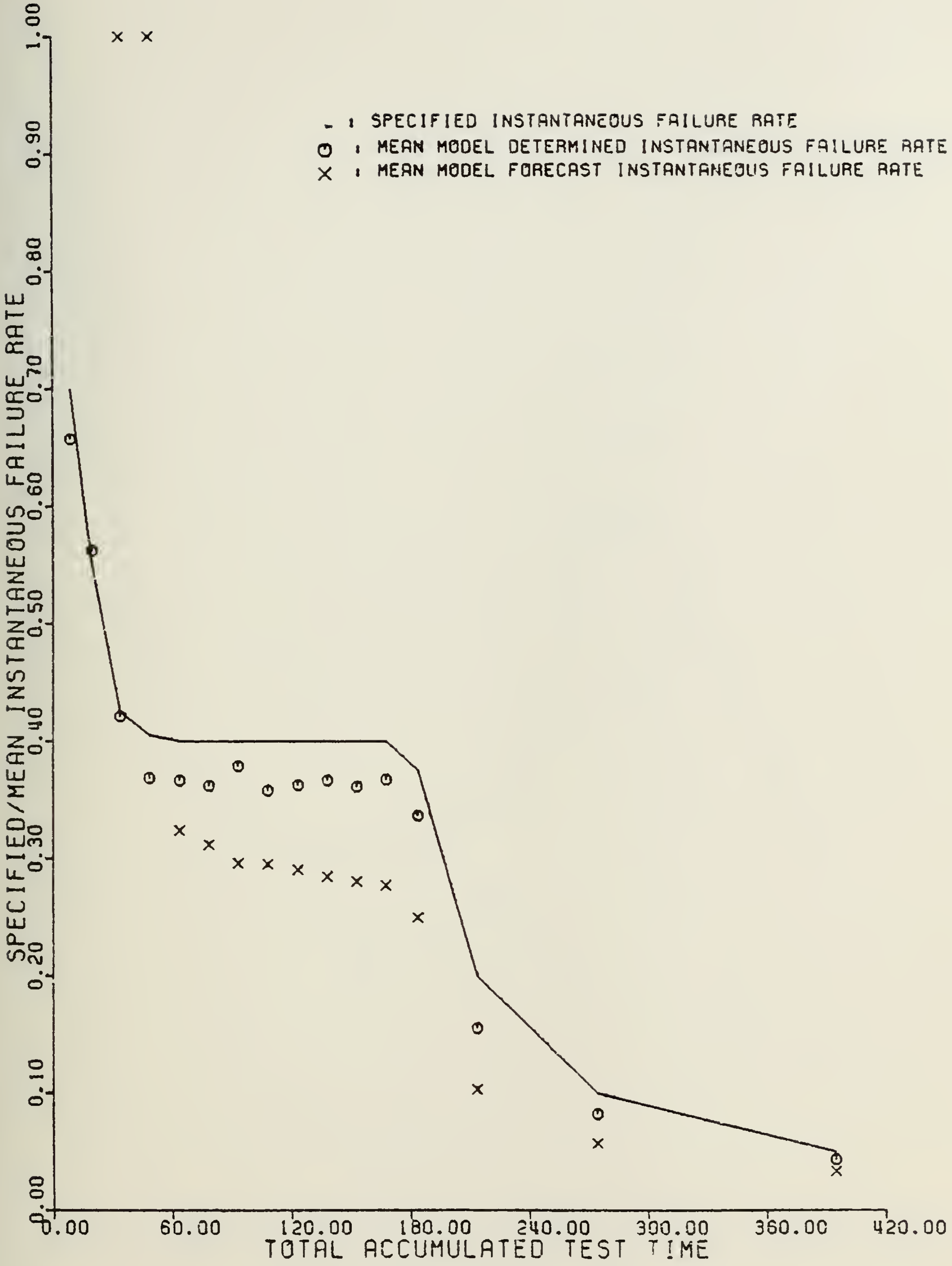

FIGURE 3.54

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 12: 16 PHASES, 20 TESTS/PHASE

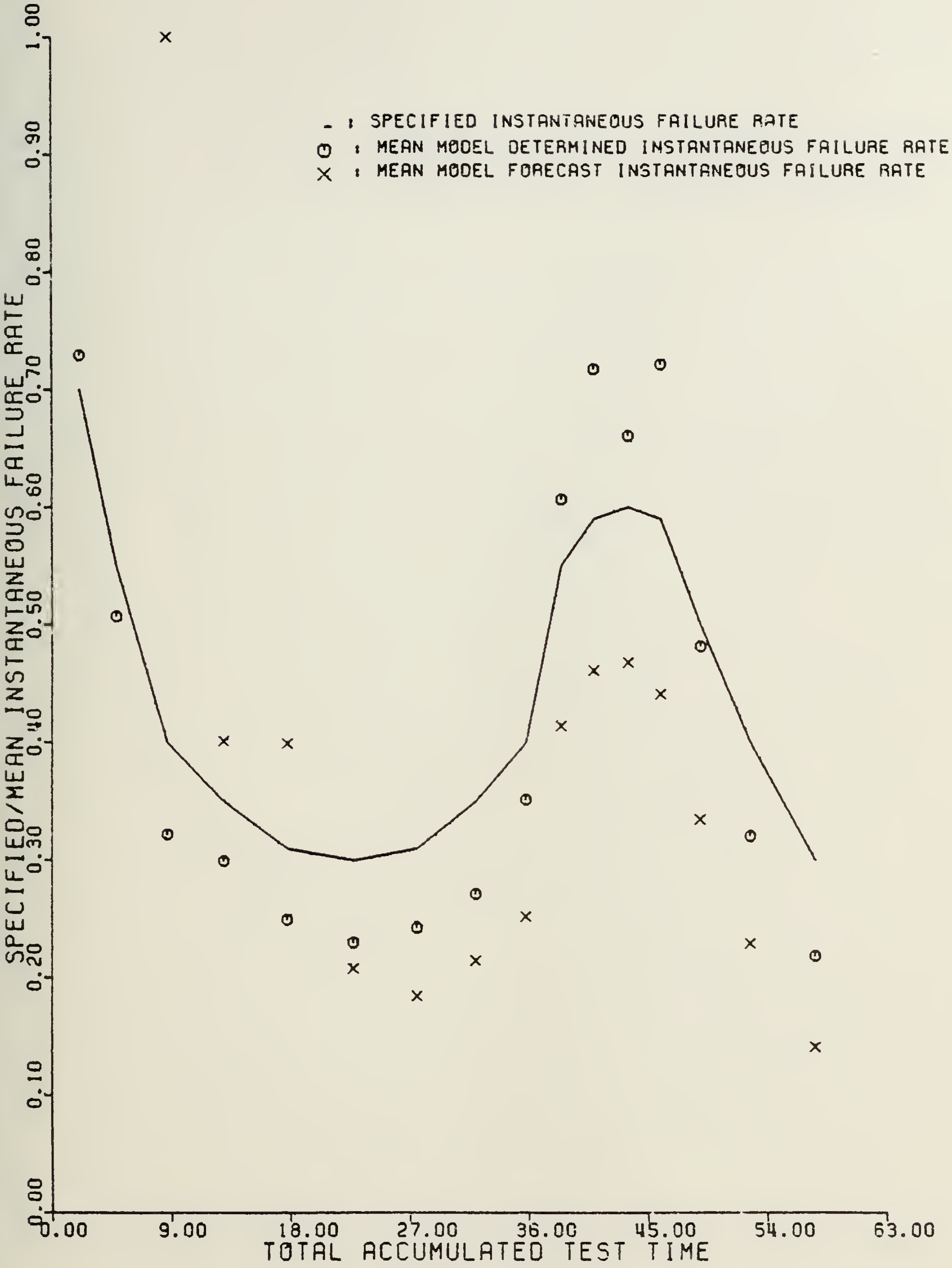

FIGURE 3.55

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 13: 16 PHASES, 5 TESTS/PHASE

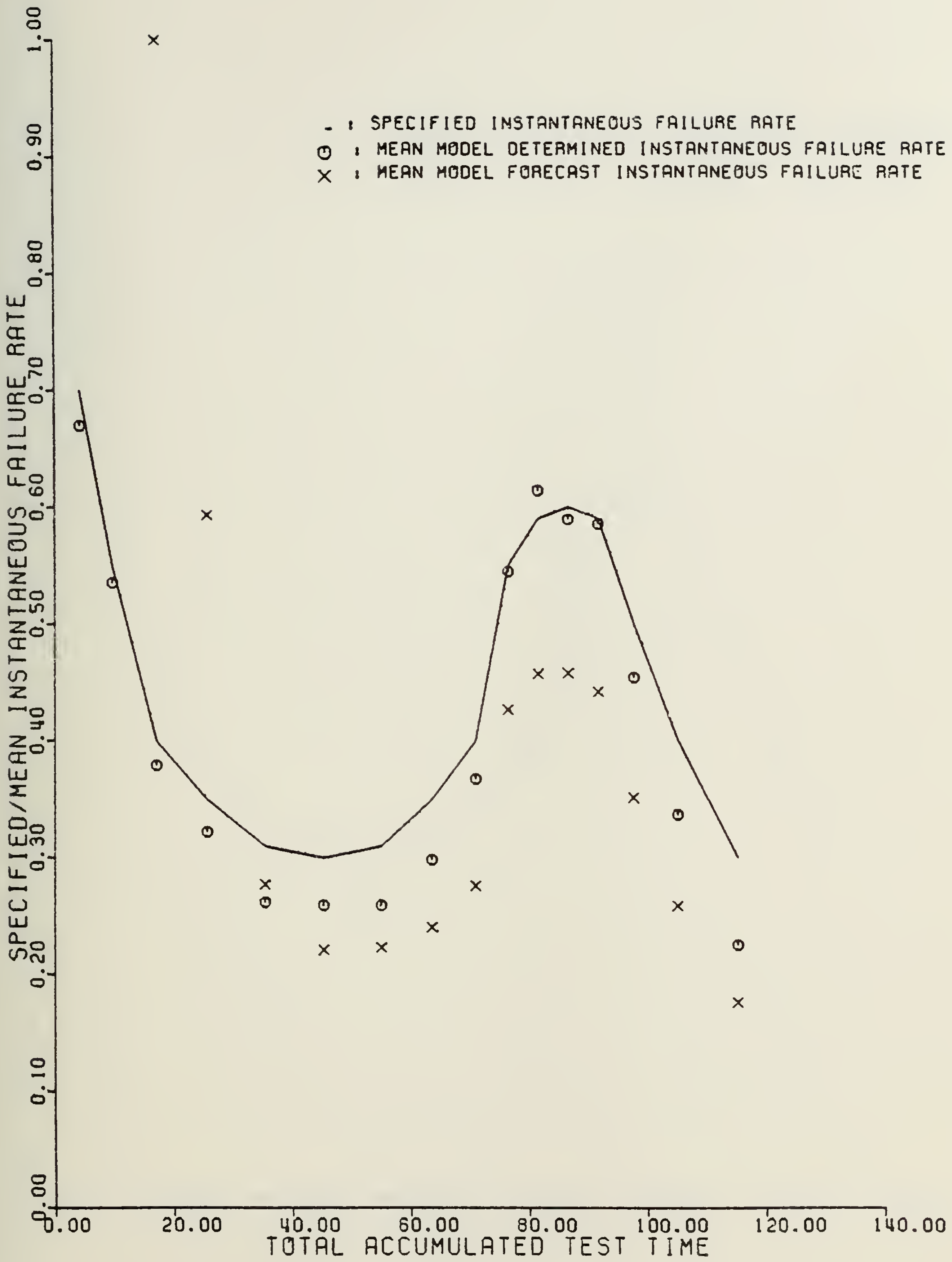

FIGURE 3.56

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 13: 16 PHASES, 10 TESTS/PHASE

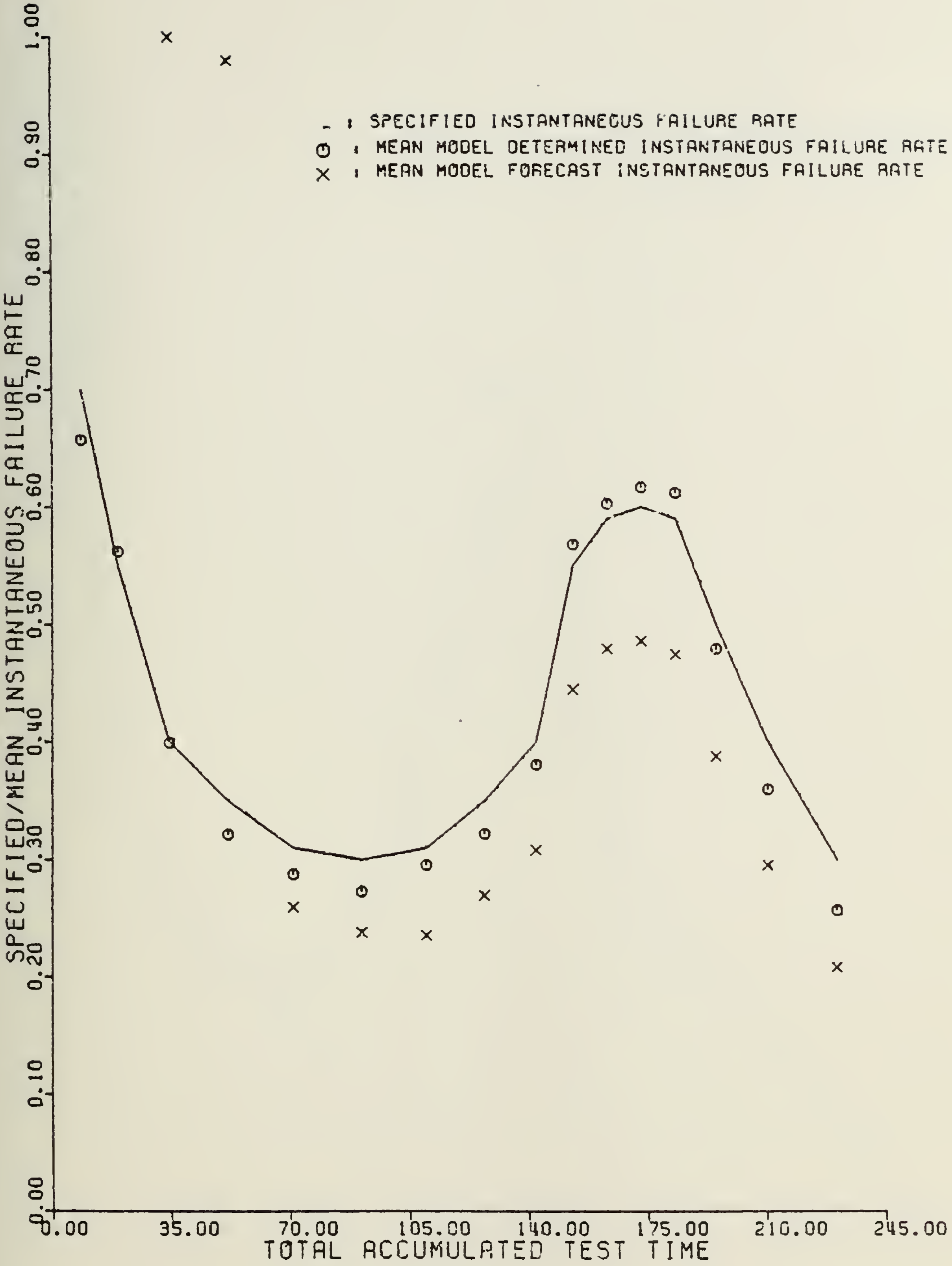

FIGURE 3.57

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 13: 16 PHASES, 20 TESTS/PHASE

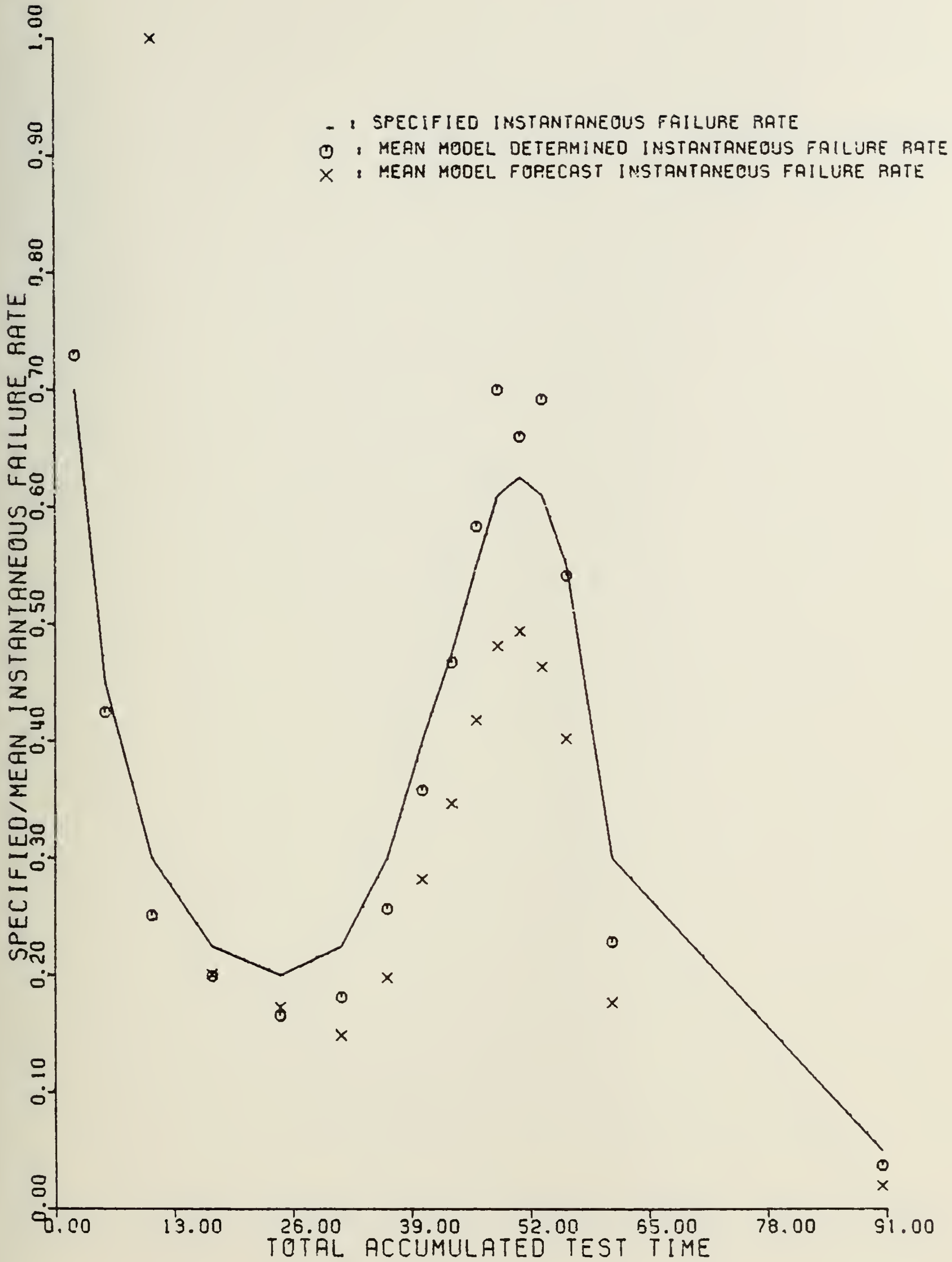

FIGURE 3.58

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 14: 16 PHASES, 5 TESTS/PHASE

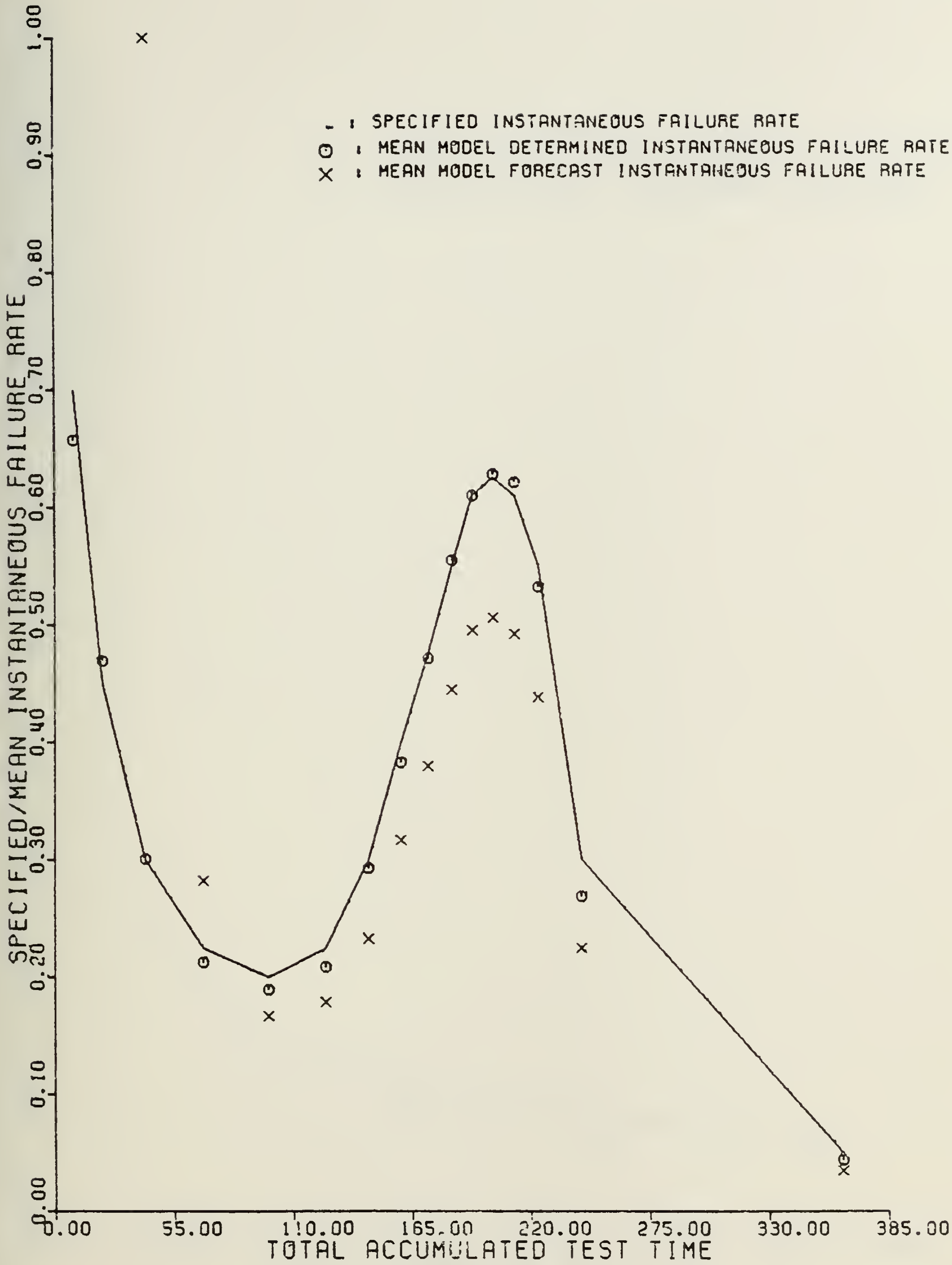

FIGURE 3.59
INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET 14: 16 PHASES, 20 TESTS/PHASE

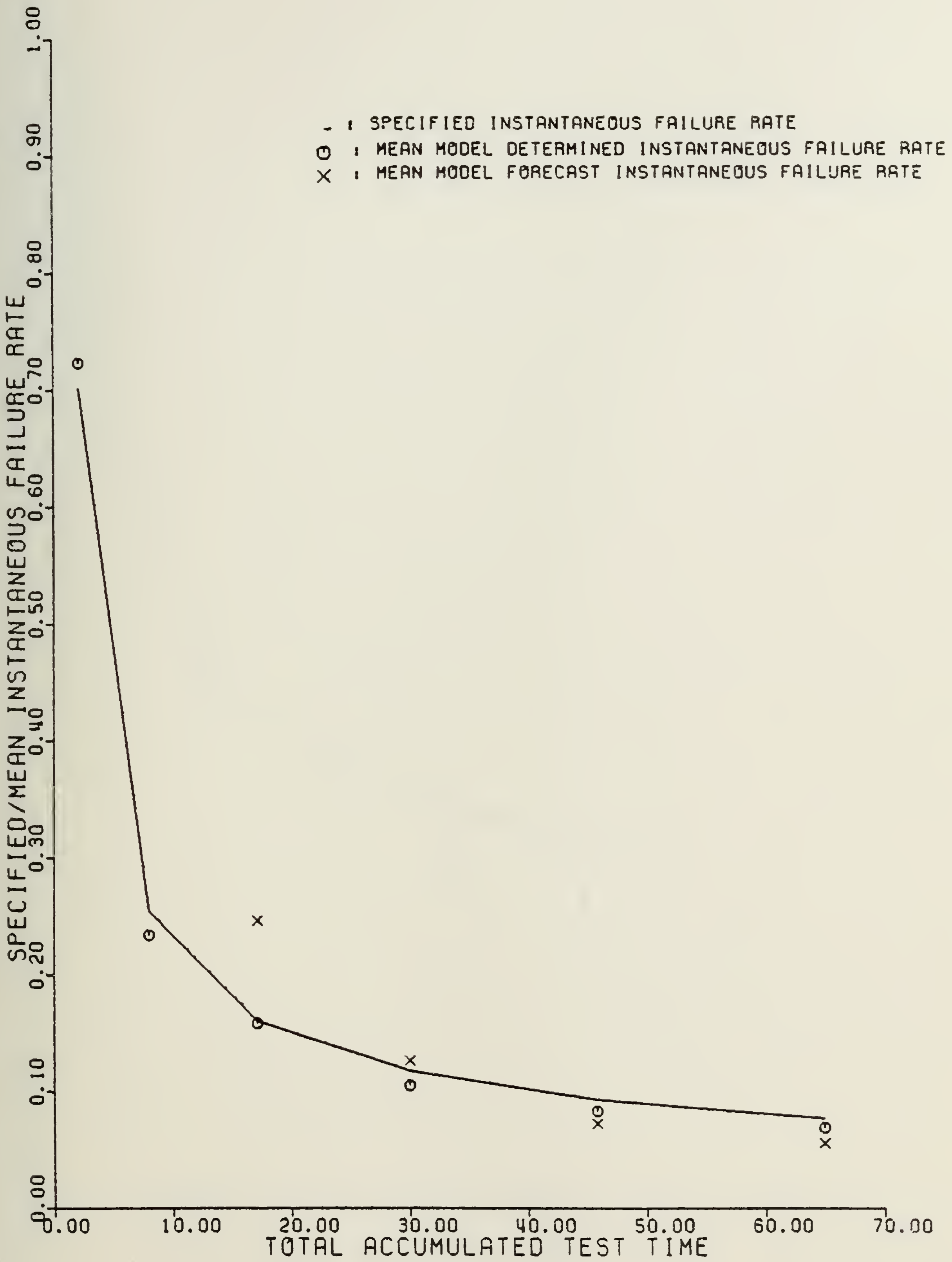

FIGURE 3.60

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD1: 6 PHASES, 5 TESTS/PHASE

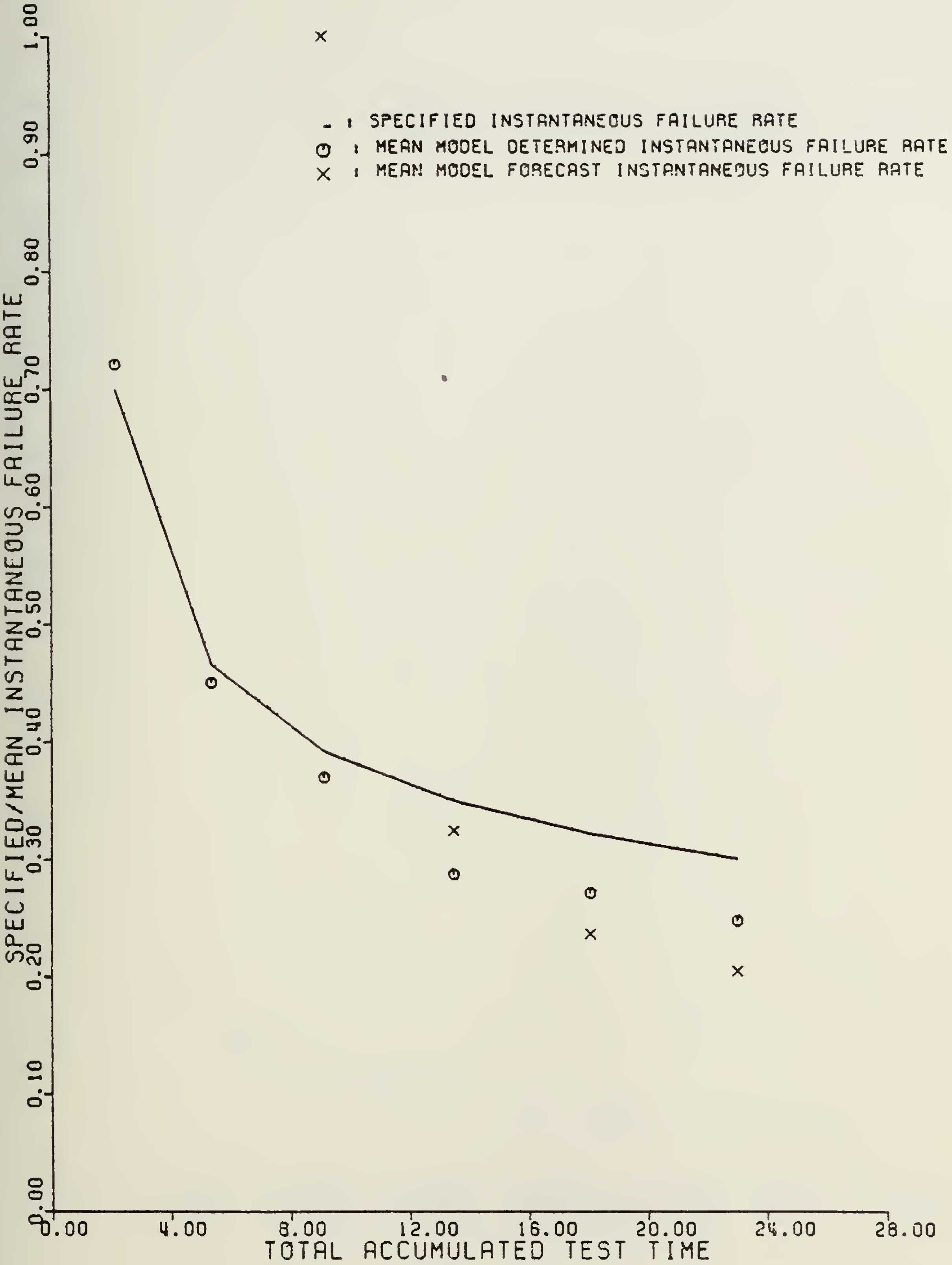

FIGURE 3.61
INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD2: 6 PHASES, 5 TESTS/PHASE

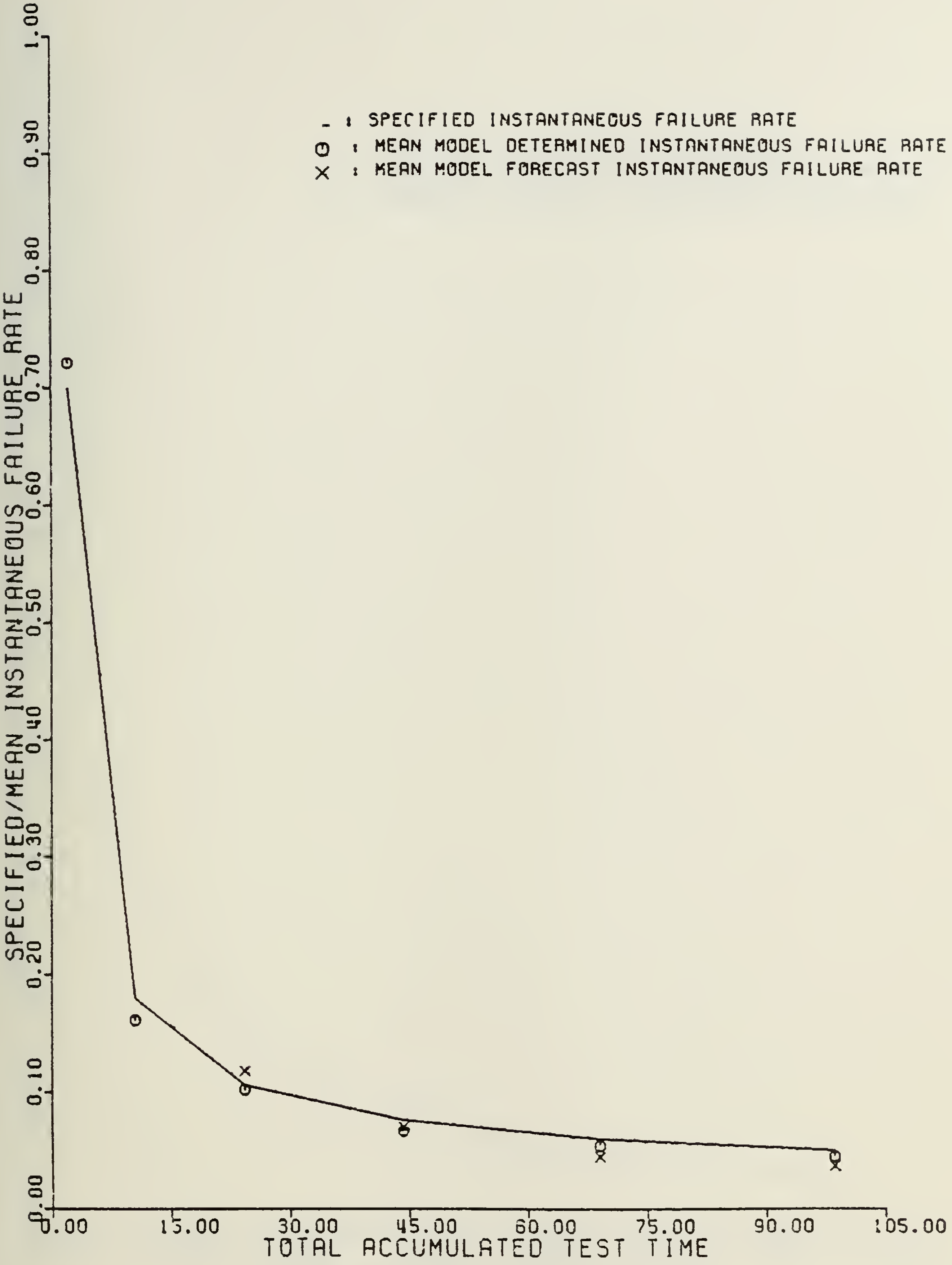

FIGURE 3.62
INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD3: 6 PHASES, 5 TESTS/PHASE

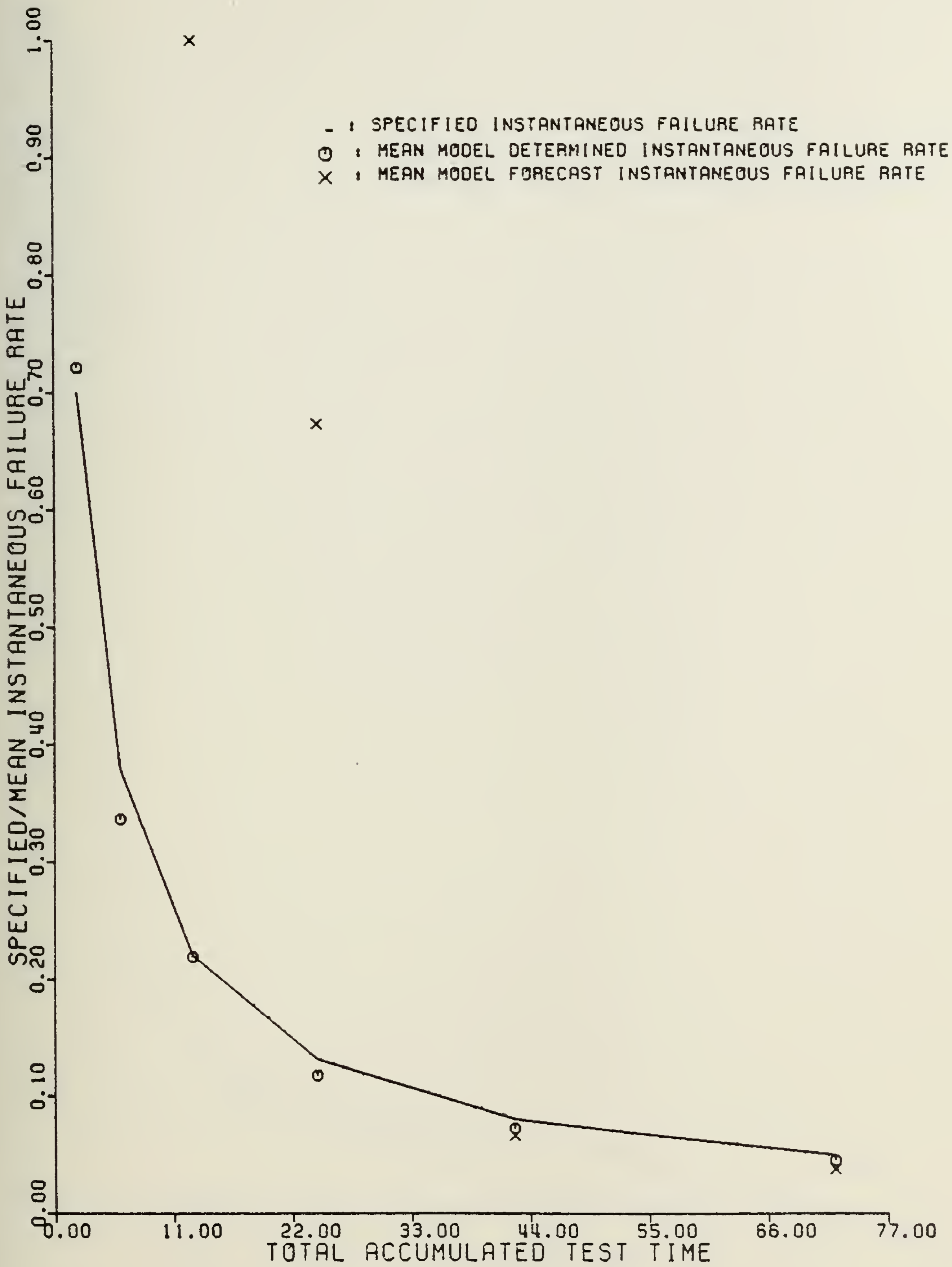

FIGURE 3.63

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD4: 6 PHASES, 5 TESTS/PHASE

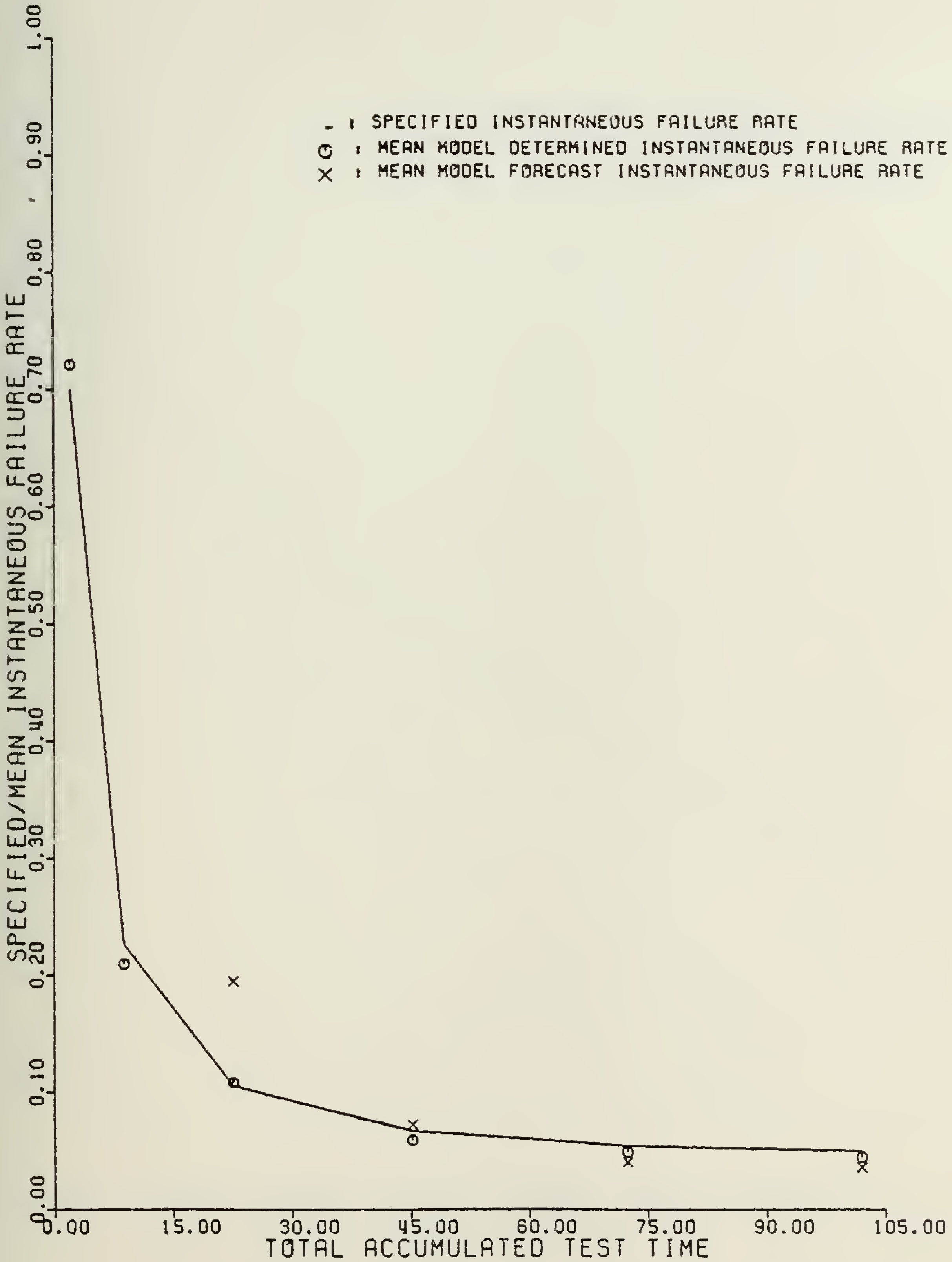

FIGURE 3.64

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD5: 6 PHASES, 5 TESTS/PHASE

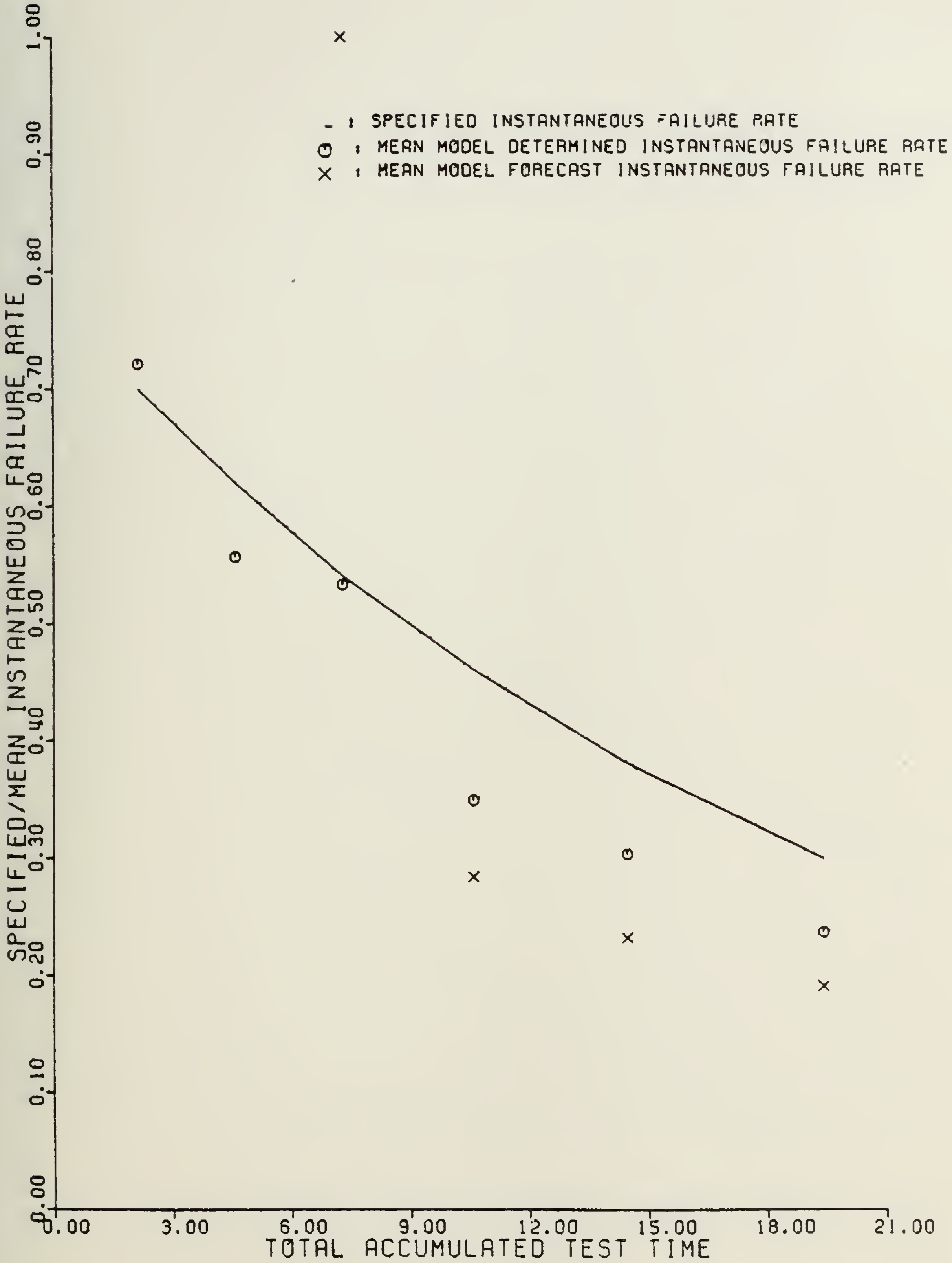

FIGURE 3.65

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD6: 6 PHASES, 5 TESTS/PHASE

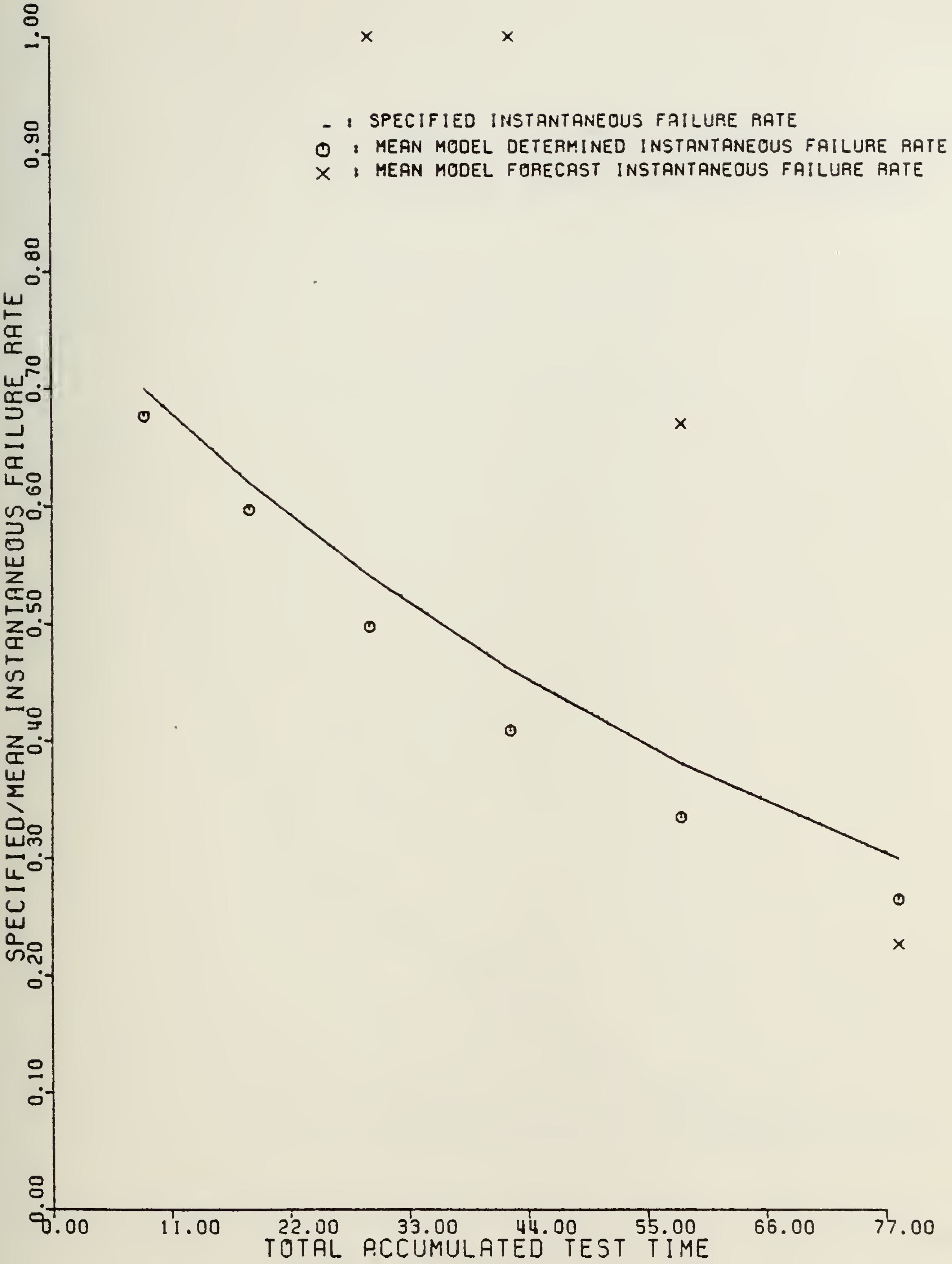

FIGURE 3.66

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD6: 6 PHASES, 20 TESTS/PHASE

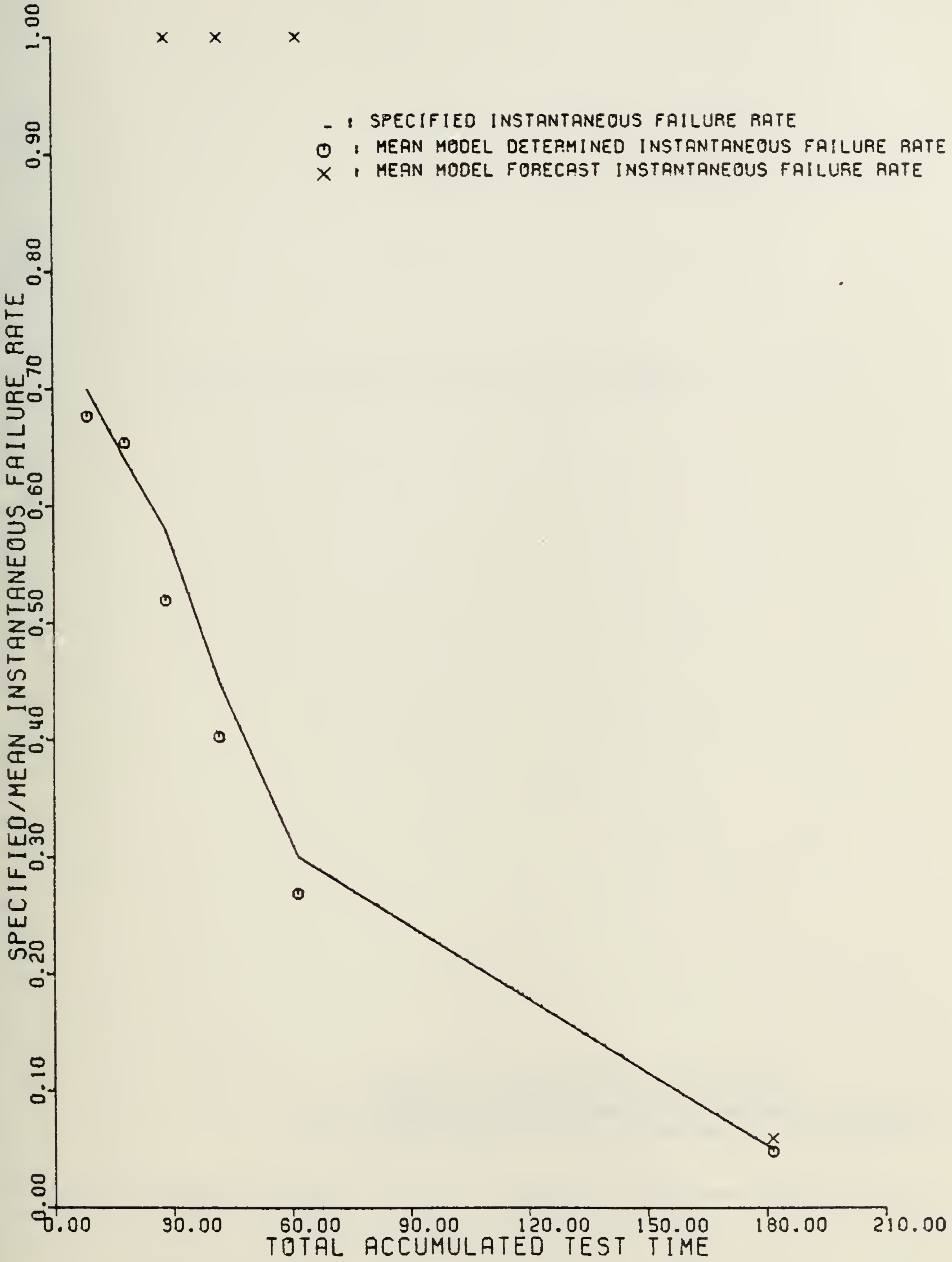

FIGURE 3.67

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD7: 6 PHASES, 20 TESTS/PHASE

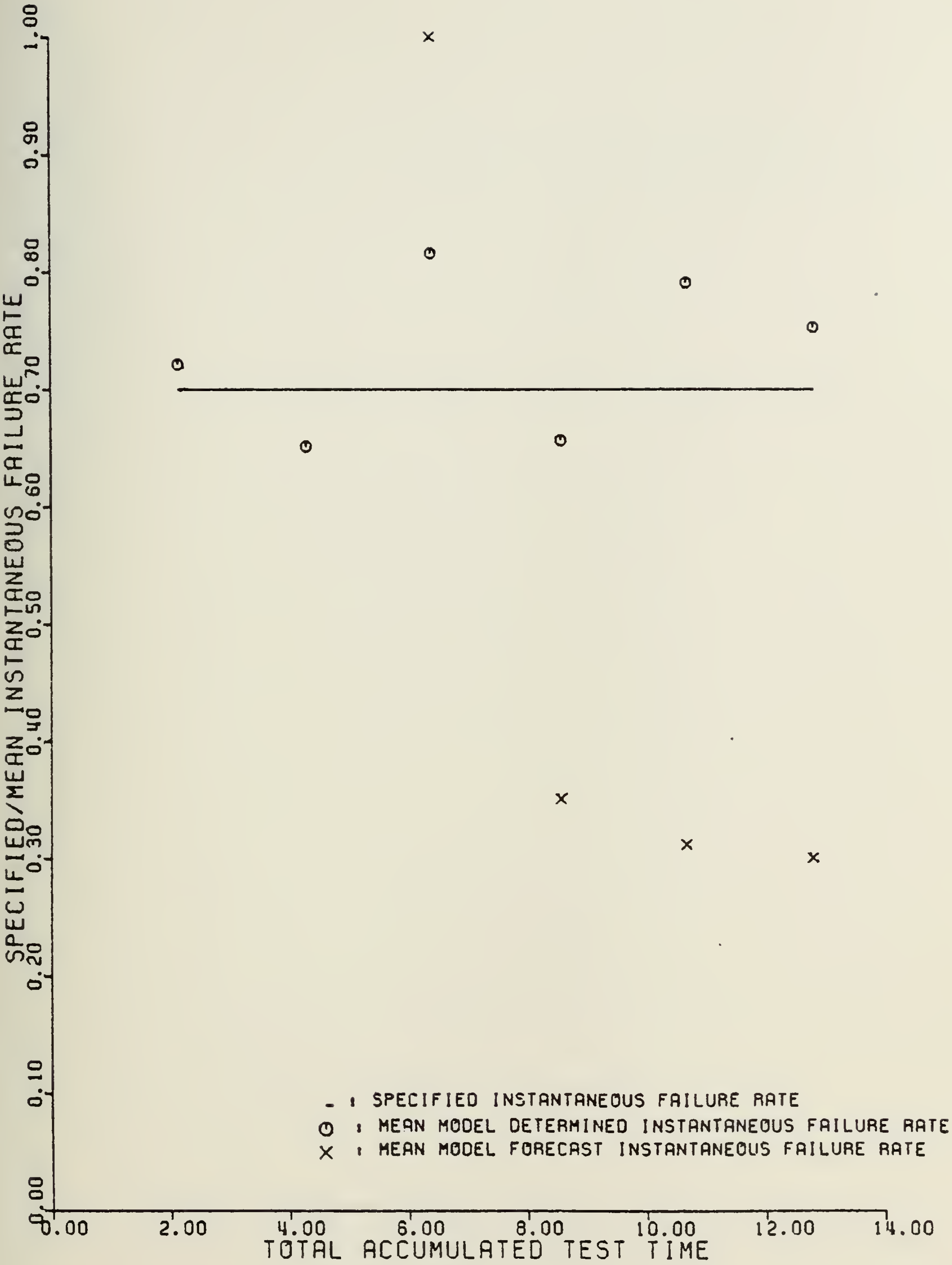

FIGURE 3.68

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD8: 6 PHASES, 5 TESTS/PHASE

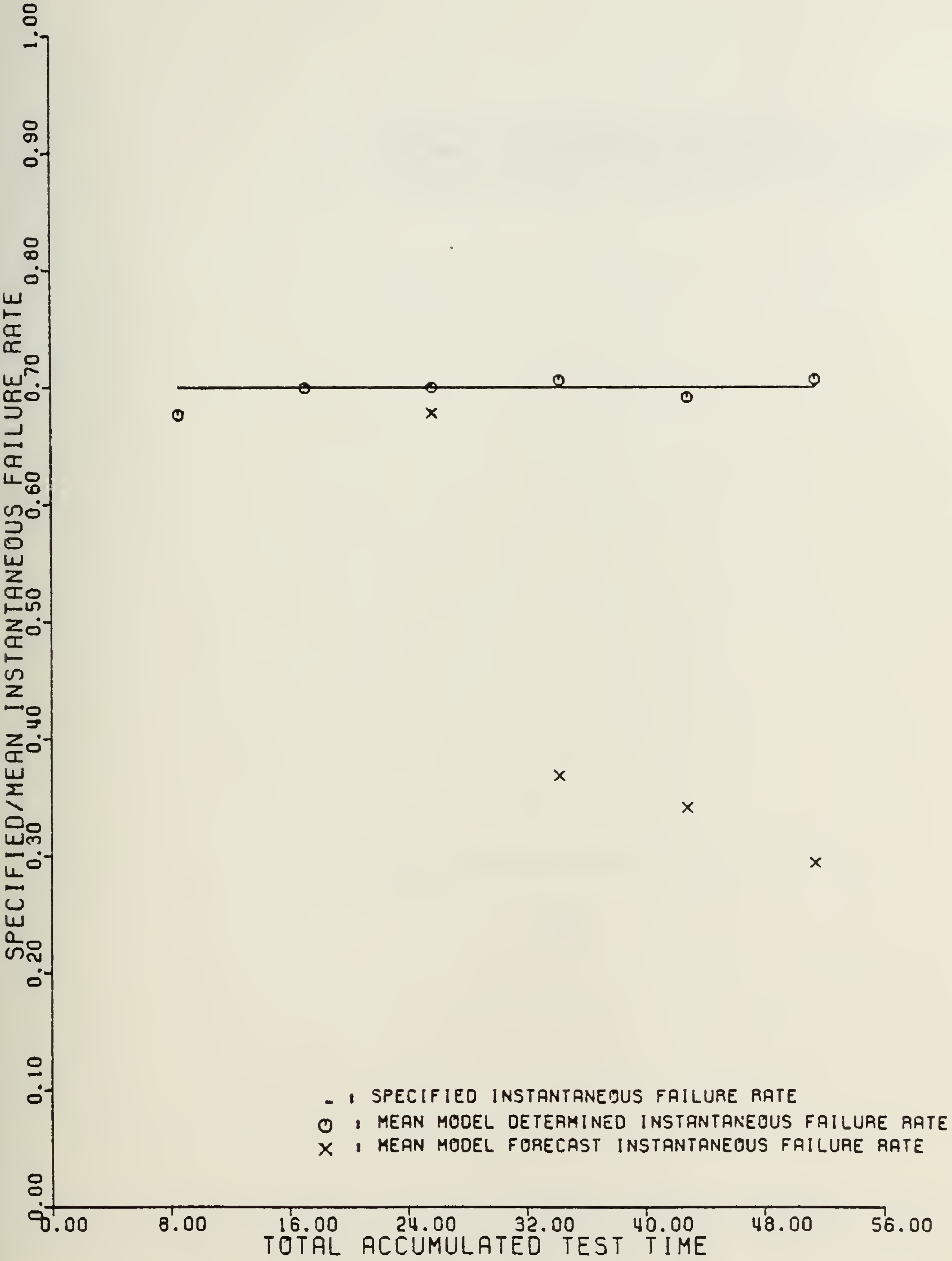

FIGURE 3.69

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD8: 6 PHASES, 20 TESTS/PHASE

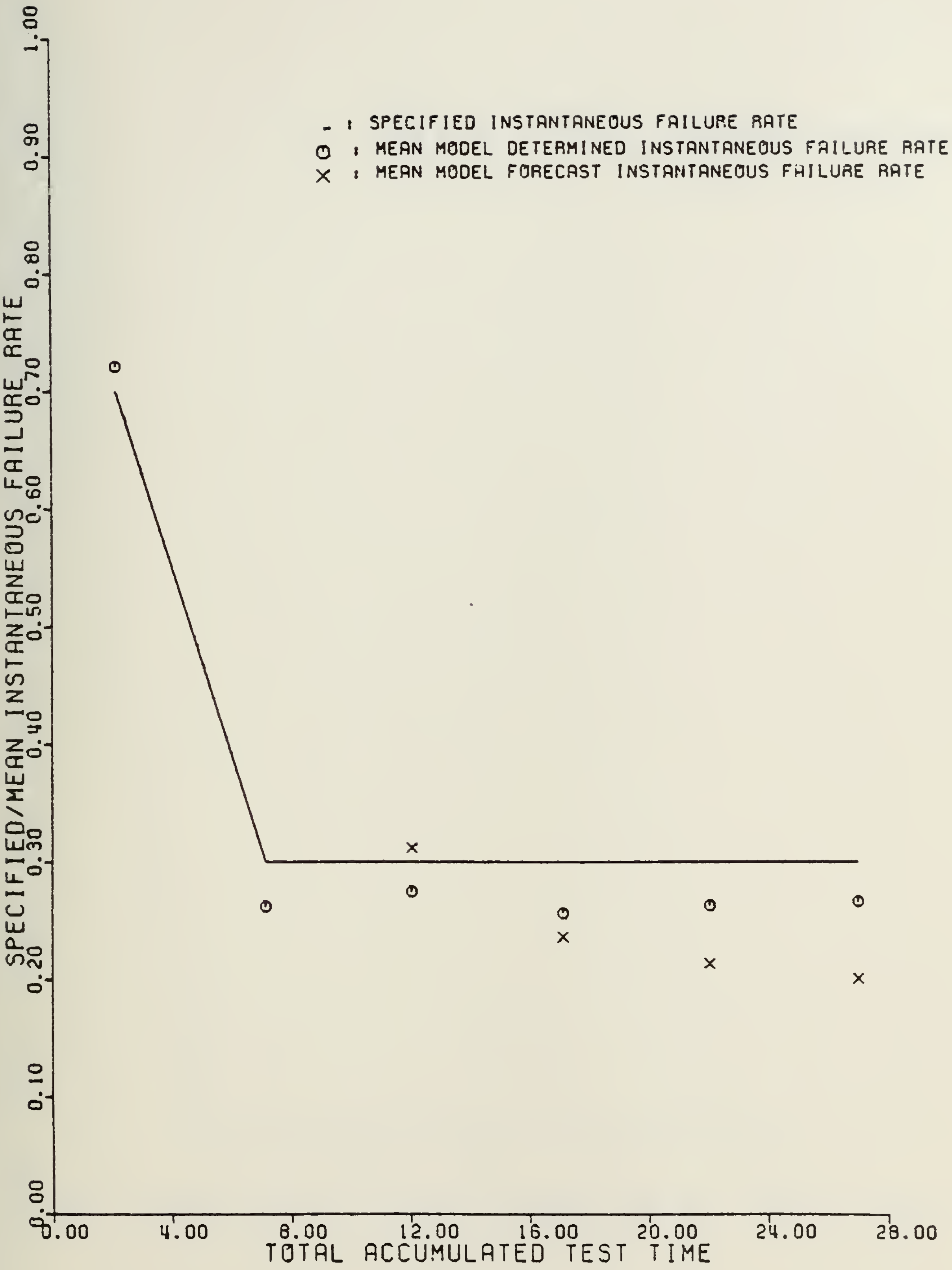

FIGURE 3.70

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD9: 6 PHASES, 5 TESTS/PHASE

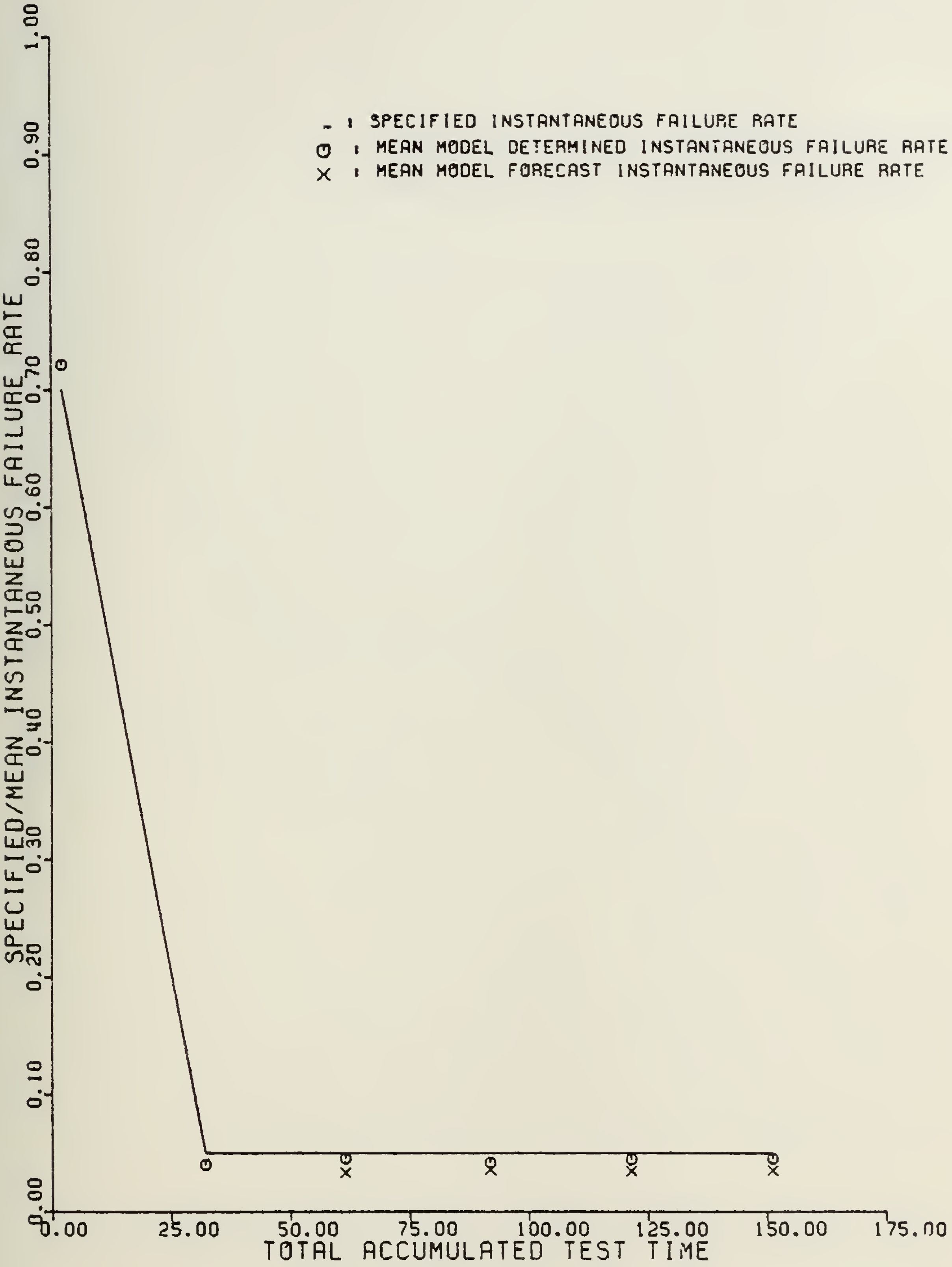

FIGURE 3.71

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD10: 6 PHASES, 5 TESTS/PHASE

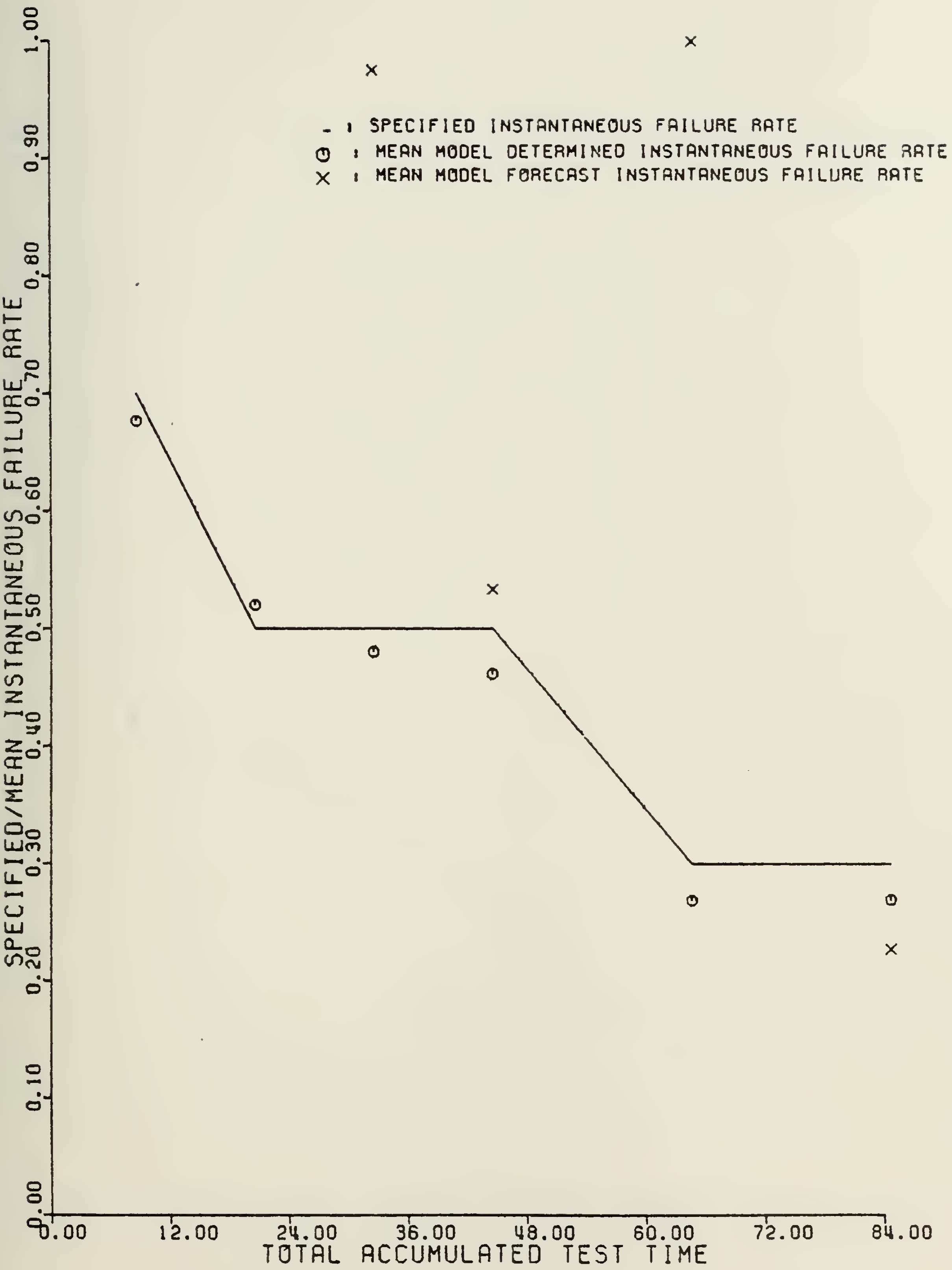

FIGURE 3.72

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD11: 6 PHASES, 20 TESTS/PHASE

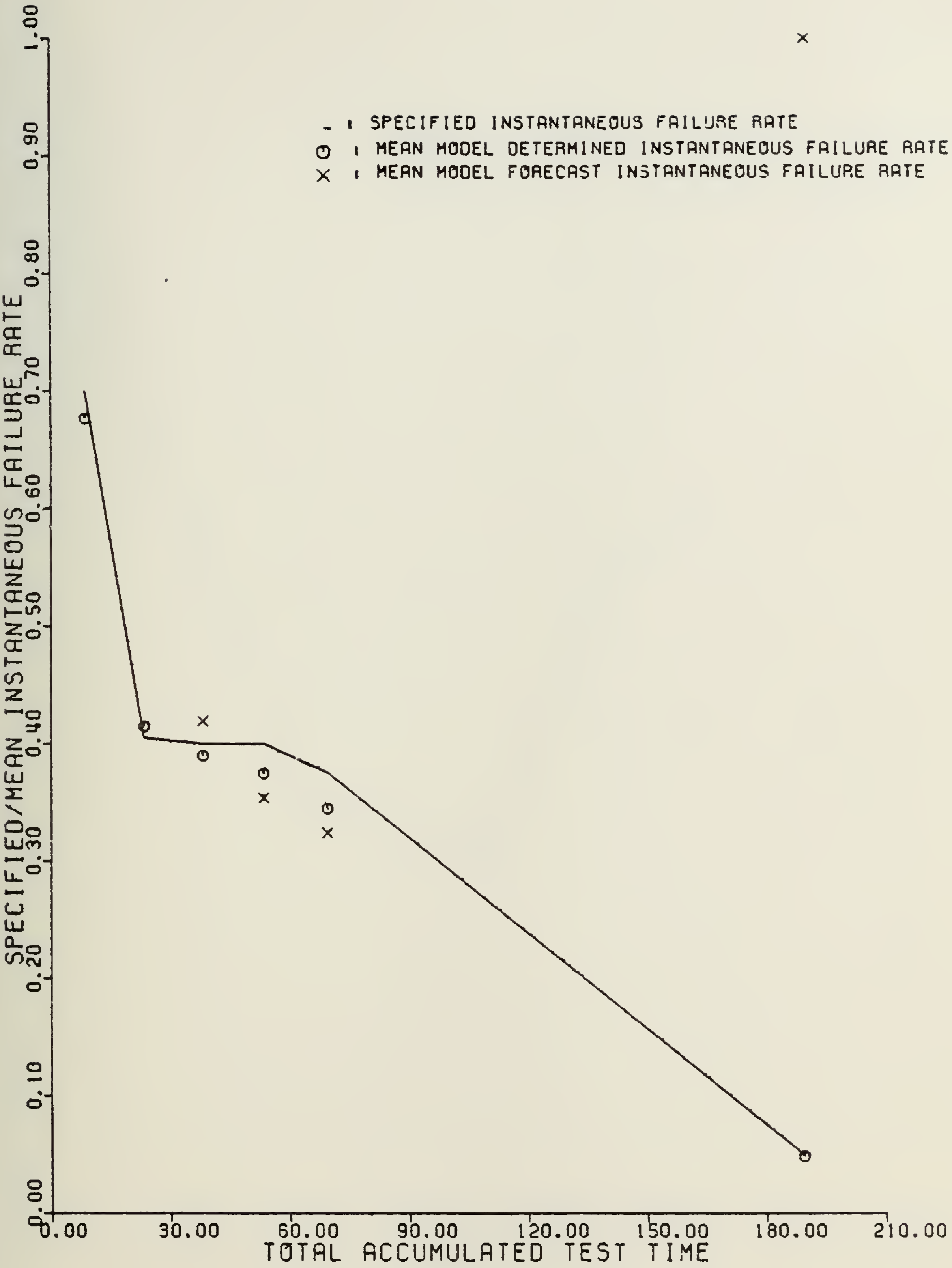

FIGURE 3.73

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD12: 6 PHASES, 20 TESTS/PHASE

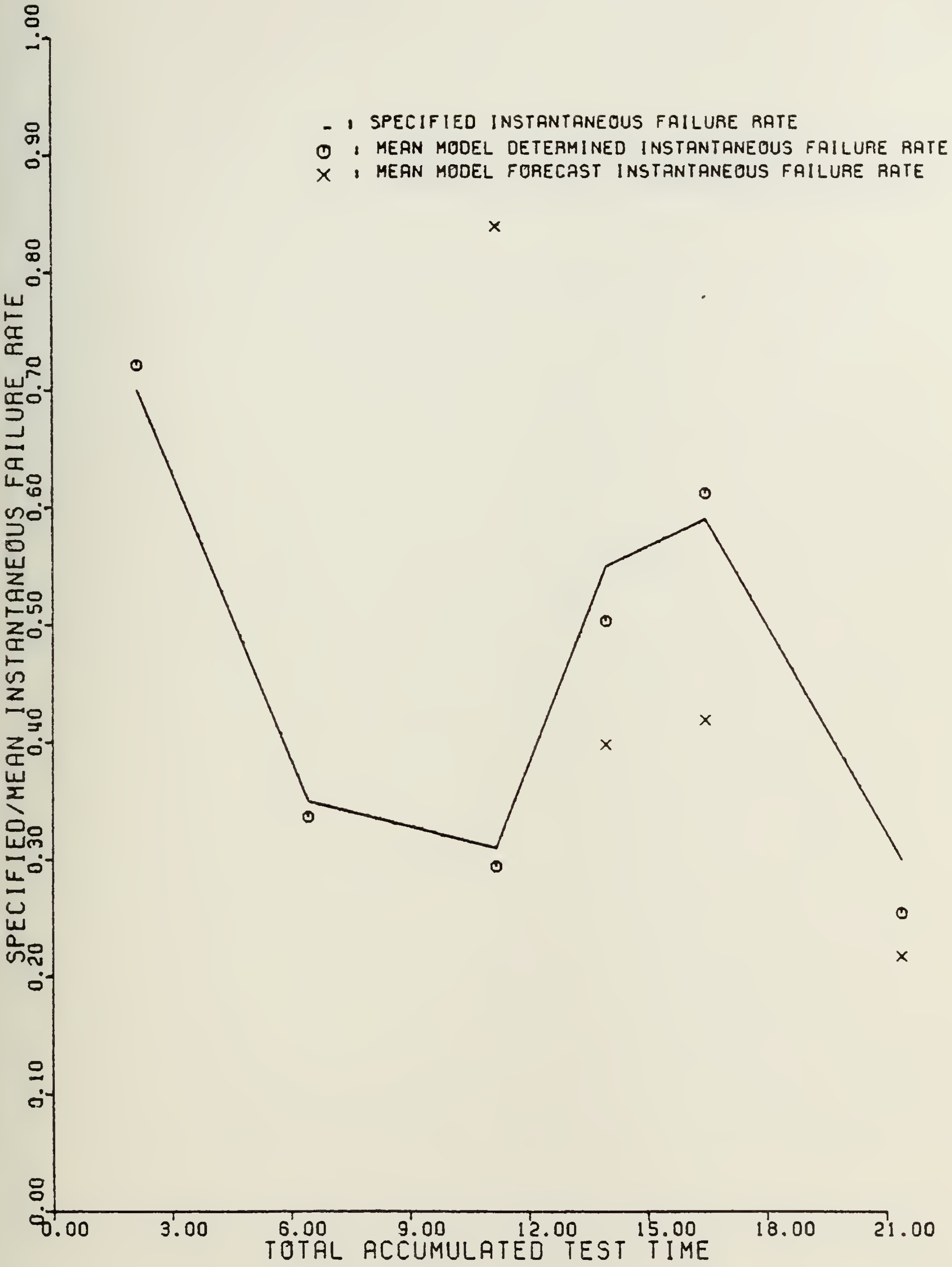

FIGURE 3.74

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD13: 6 PHASES, 5 TESTS/PHASE

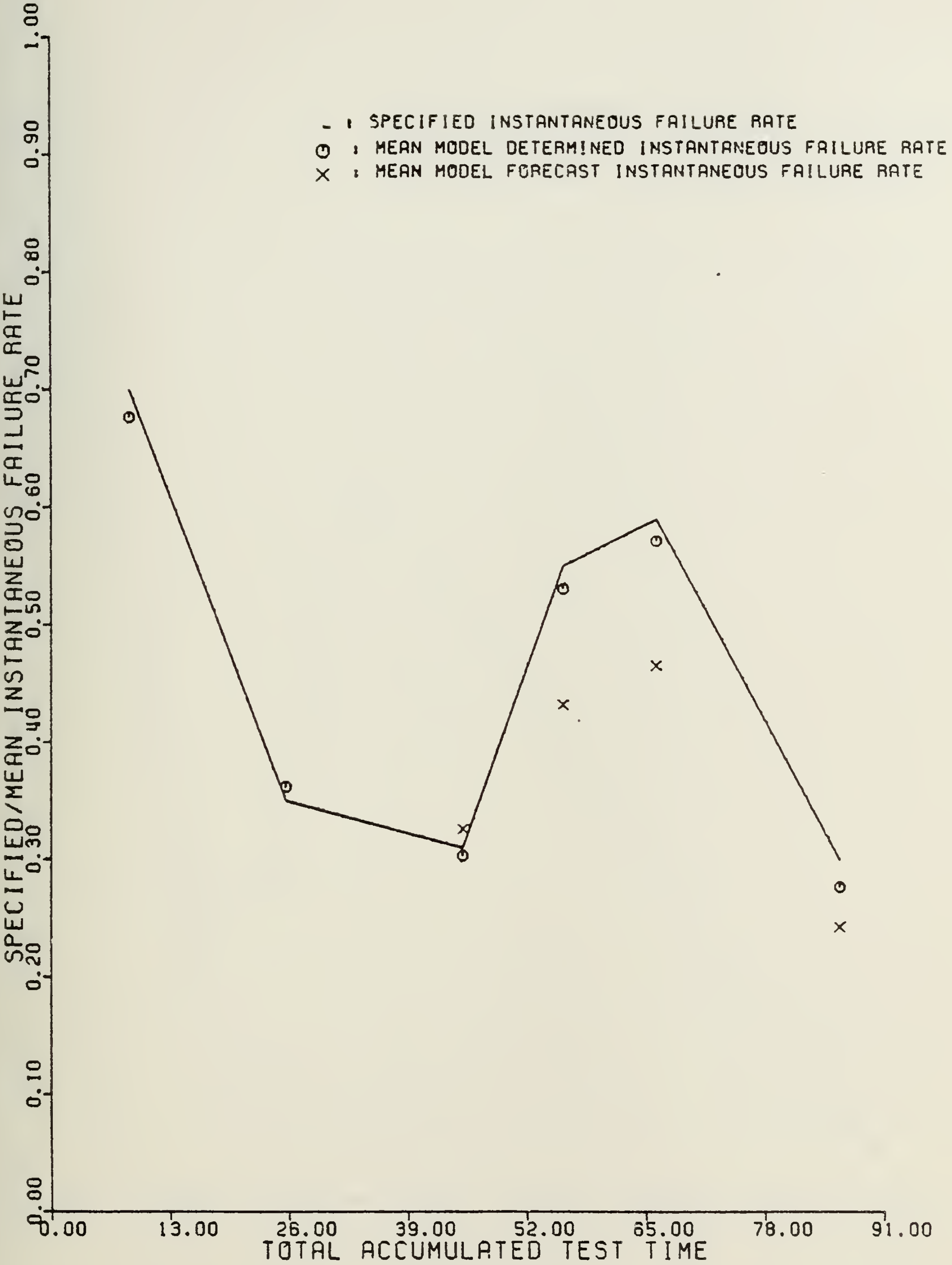

FIGURE 3.75

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD13: 6 PHASES, 20 TESTS/PHASE

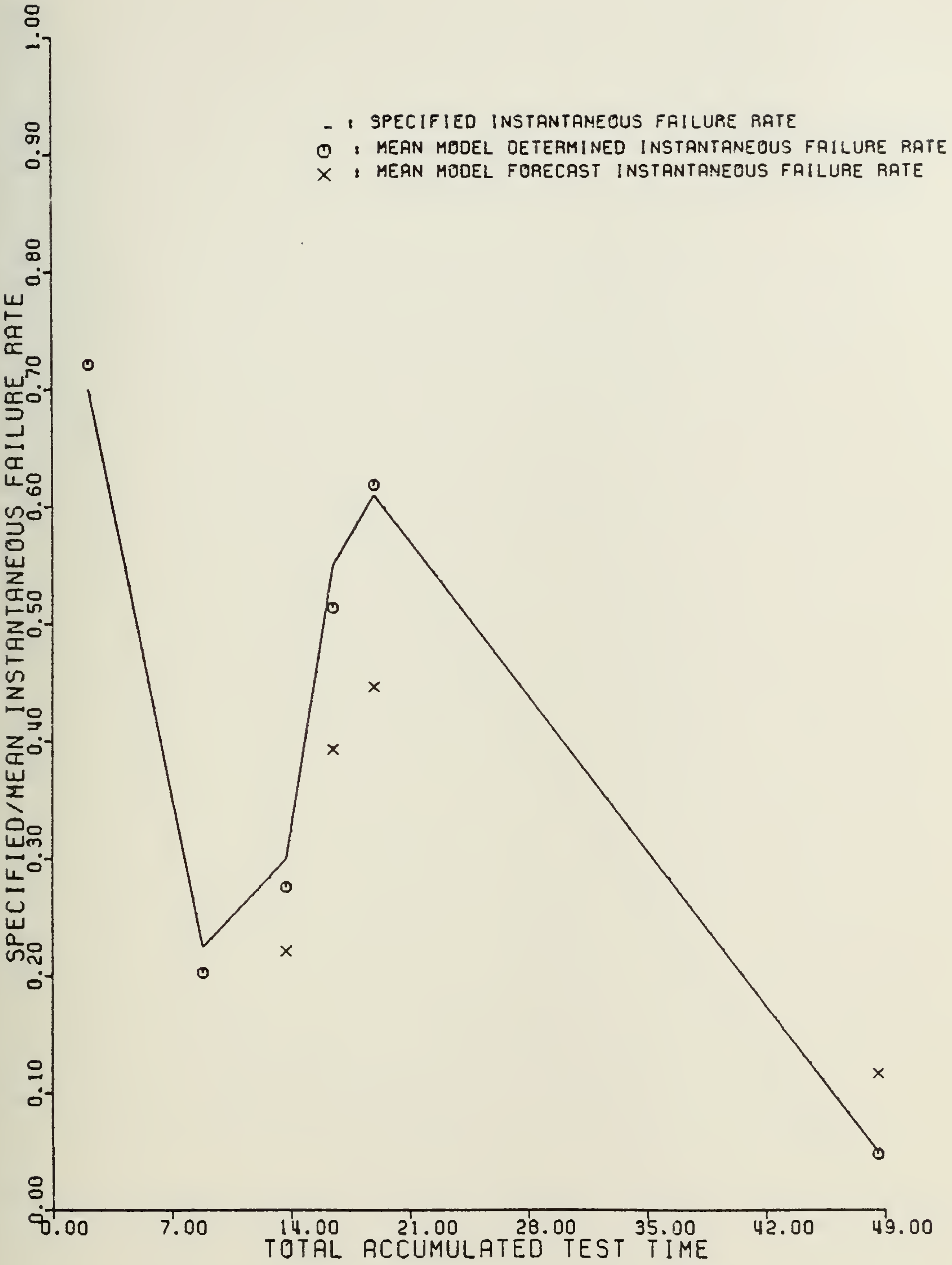

FIGURE 3.76

INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD14: 6 PHASES, 5 TESTS/PHASE

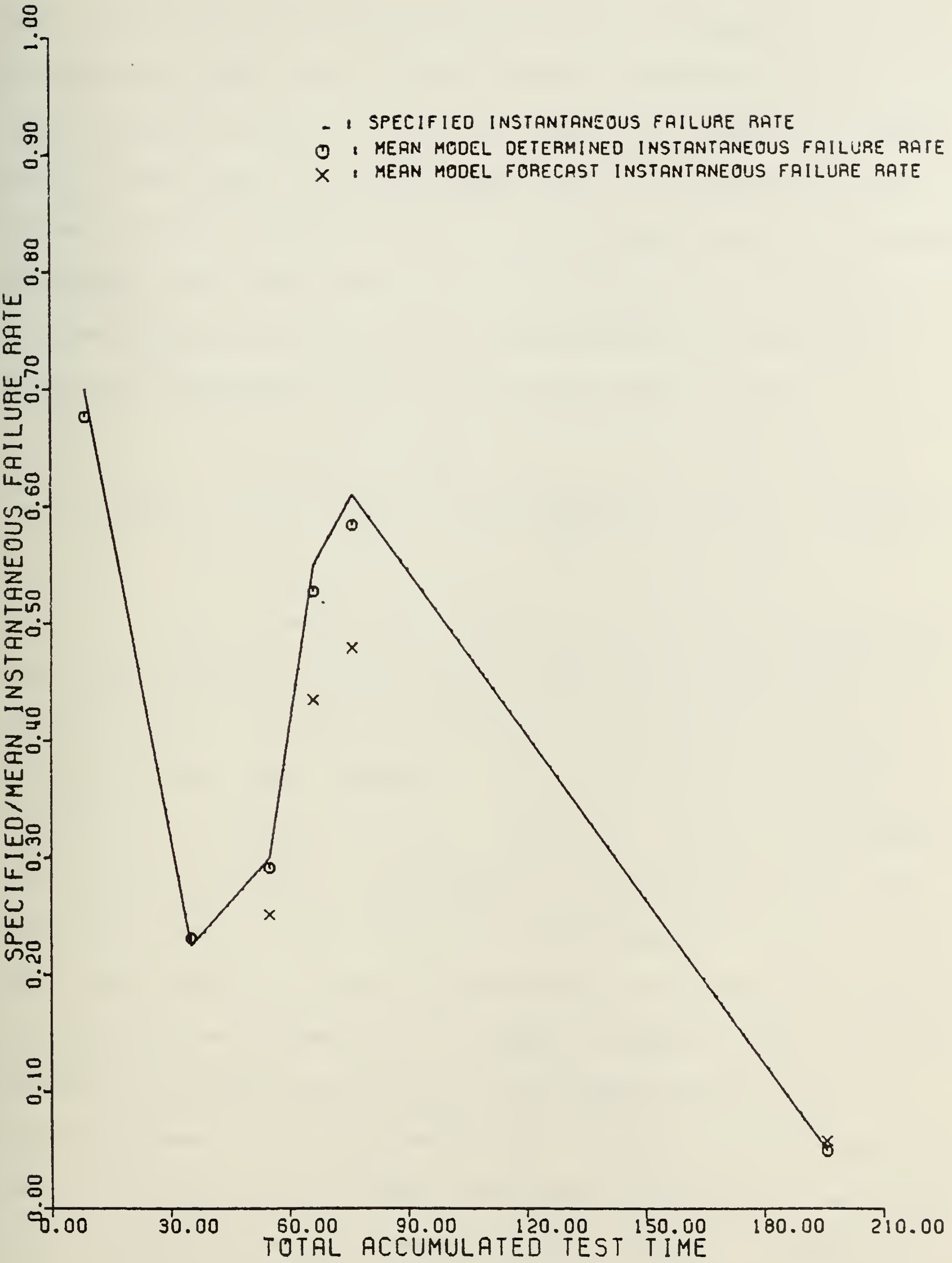

FIGURE 3.77
INSTANTANEOUS RELIABILITY GROWTH MODEL PERFORMANCE
LAMBDA SET MOD14: 6 PHASES, 20 TESTS/PHASE

these tables are the percentage standard errors as computed in equations 3.48 and 3.53. From figure 3.47 the mean model determined instantaneous failure rate was read as 0.24 for the sixteenth test phase of the lambda set 6, 16 test phase, 20 tests per phase reliability testing procedure. In table 3.17 the percentage standard error corresponding to the sixteenth test phase of the lambda set 6, NT_i = 20 simulations is 24%. By equation 3.48 the standard deviation of the instantaneous failure rate model's determined instantaneous failure rate for this phase is then

$$\text{S.D.}\ \widehat{\lambda}_{T_{16}} = \frac{\text{P.S.E.}\ \widehat{\lambda}_{T_{16}} \times \overline{\widehat{\lambda}}_{T_{16}}}{100} = \frac{24 \times 0.24}{100} = 0.0576\ . \qquad (3.55)$$

A quick examination of tables 3.15 thru 3.26 reveals that the goal of 30% percentage standard error of failure rate estimates utilized in evaluating the variability performance of the cumulative failure rate reliability growth model (chapter III.A.5.b.) will be too restrictive for evaluating the instantaneous failure rate model's variability performance. Also, unlike the variability performance of the cumulative failure rate model which almost uniformly improved as test phases of the testing procedure were completed, variability performance of the instantaneous model oscillates for many lambda sets. For example, keeping in mind that model variability performance begins with the percentage standard errors recorded for phase 2, the instantaneous model's variability performance for lambda set 1, five tests per phase, determined failure rate in table 3.15 improves steadily from 76% in phase 2 to 38% in phase 11. In phase 12 the model's variability performance jumps up to 41%, improves back to 32% by phase 14, and finally, climbs to 35% for the last test phase, phase 16.

This oscillation in the variability performance of the instantaneous failure rate model means that even if a reasonable percentage standard error benchmark is selected, determining the first phase for which variability performance on all lambda sets is within the goal does not guarantee the performance goal will be satisfied for all subsequent test phases of the acquisition cycle.

Tables 3.15, 3.16, and 3.17 display the instantaneous failure rate reliability growth model's variability performance for determined instantaneous failure rate estimates for the sixteen test phase procedures. If a percentage standard error goal of 40% is set, the goal is never achieved for $NT_i = 5$ or $NT_i = 10$ (tables 3.15 and 3.16), but is achieved for $NT_i = 20$ (table 3.17) on test phases 5, 7, 8, 14, and 16. Again, the oscillating character of the instantaneous model's variability performance is displayed by this method of examination. Tables 3.18, 3.19, and 3.20 show that in the case of the contracted six test phase procedures the instantaneous failure rate model's variability performance for determined instantaneous failure rate fails to attain the 40% goal for all simulated tests per phase specifications; i.e., $NT_i = 5$, 10, and 20.

In both the sixteen and six test phase reliability testing procedures determined failure rate estimates given by the instantaneous model were least accurate for lambda set 8 and MOD8. See figures 3.49, 3.68, and 3.69. Also, the model's accuracy performance for these lambda sets was the only case in which no definite bias trend was evident. The instantaneous failure rate model's variability performance for determined instantaneous failure rate estimates is almost uniformly the worst for lambda sets 8 and MOD8 as evidenced in tables 3.15 thru 3.20. This was

to be expected based on the model's accuracy performance for these particular specified underlying instantaneous failure rate paths.

Tables 3.21 thru 3.26 contain variability performance results of the instantaneous model's forecast failure rates for both the sixteen and six test phase reliability testing procedures. Percentage standard error entries of 1000% indicate that the actual percentage standard errors in these cases were greater than or equal to 1000%. As with the cumulative failure rate model, lambda set 7, 12, 14, MOD7, MOD12, and MOD14 provide difficulty for the instantaneous model. These lambda sets are the ones which take the most rapid "plunge" to a low specified instantaneous failure rate of $\lambda_{16} = 0.0500$ or $\lambda_6 = 0.0500$. Again, if these lambda sets are not considered, then a 40% percentage standard error goal is achieved on phase 14 for $NT_i = 5$ and 10 and on phase 11 for $NT_i = 20$ in the case of the sixteen phase reliability testing procedure simulations. The instantaneous model's variability performance for forecasting during six test phase cycles is very poor achieving the 40% goal on the last test phase of the "most data" case ($NT_i = 20$) for only seven of the fourteen lambda sets.

C. DISCRETE RELIABILITY GROWTH MODEL

1. Model Description

The discrete reliability growth model evaluated is of the form

$$R_m = 1 - \frac{1}{\exp(\alpha + \beta m)} \qquad (3.56)$$

where R_m = component reliability after the m^{th} modification of the component

m = number of modifications that have been made to the component, and

α, β = constants that determine the change in the model estimated component reliability as modifications to the component are accomplished.

TABLE 3.15

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

16 PHASES, 5 TESTS/PHASE, DETERMINED FAILURE RATE

PHASE *****	LAMBDA SET ********** 1 ****	2 ****	3 ****	4 ****	5 ****	6 ****	7 ****	8 ****	9 ****	10 ****	11 ****	12 ****	13 ****	14 ****
1	62	62	62	62	62	62	62	62	62	62	62	62	62	62
2	76	79	80	77	80	84	77	81	81	96	75	79	79	76
3	67	67	62	64	68	78	78	76	62	62	67	66	66	64
4	68	99	66	86	70	108	106	99	67	56	97	84	77	69
5	49	70	49	54	49	92	93	86	52	40	78	60	54	50
6	47	52	46	47	47	77	79	77	41	36	61	49	45	42
7	47	50	46	51	49	66	70	72	42	38	58	47	44	38
8	45	49	44	50	48	67	73	81	42	35	64	48	40	38
9	43	47	43	49	47	65	71	84	42	34	66	48	40	42
10	40	41	39	48	43	63	76	78	39	29	66	45	35	40
11	38	46	37	42	38	58	54	82	46	31	57	54	48	46
12	41	59	39	45	40	70	60	100	61	36	63	72	60	58
13	38	70	37	41	37	86	61	126	74	38	67	91	67	54
14	32	39	31	38	31	43	45	88	41	25	40	48	40	36
15	33	39	31	39	31	42	51	75	40	26	39	63	36	31
16	35	50	34	40	33	48	68	99	58	32	46	60	45	72

TABLE 3.16

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

16 PHASES, 10 TESTS/PHASE, DETERMINED FAILURE RATE

	LAMBDA SET													
	**********													
PHASE	1	2	3	4	5	6	7	8	9	10	11	12	13	14
*****	****	****	****	****	****	****	****	****	****	****	****	****	****	****
1	58	58	58	58	58	58	58	58	58	58	58	58	58	58
2	66	73	66	67	68	70	70	69	70	61	63	67	67	64
3	49	51	49	52	51	55	58	53	44	46	46	50	49	51
4	45	46	44	48	46	58	61	60	39	39	43	41	43	44
5	46	46	45	47	47	67	69	68	38	36	49	42	44	43
6	39	40	40	41	43	54	58	59	34	32	43	36	36	33
7	33	34	32	38	37	54	61	65	28	26	41	30	29	26
8	31	33	30	36	33	41	45	54	27	24	38	29	26	29
9	31	32	30	36	33	38	41	52	26	24	36	29	26	30
10	36	37	34	42	37	46	48	70	31	26	49	35	37	36
11	35	40	34	41	36	49	50	79	34	26	54	39	37	35
12	33	34	32	39	33	41	46	58	28	24	45	31	29	29
13	27	29	26	31	27	34	37	65	26	21	35	30	30	28
14	27	29	26	31	26	34	38	65	26	21	33	44	25	25
15	25	27	24	28	23	32	36	62	25	19	28	44	22	21
16	27	30	26	31	25	36	52	62	29	21	31	50	30	48

TABLE 3.17

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

16 PHASES, 20 TESTS/PHASE, DETERMINED FAILURE RATE

LAMBDA SET

PHASE	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	42	42	42	42	42	42	42	42	42	42	42	42	42	42
2	45	42	39	45	41	39	42	40	39	42	41	40	40	41
3	36	35	36	37	38	46	44	43	30	29	33	38	40	38
4	36	34	36	38	38	43	46	47	26	26	29	33	36	38
5	31	28	31	32	33	31	33	37	23	22	24	26	30	30
6	29	29	29	32	31	38	44	48	22	22	27	25	27	24
7	25	24	25	27	26	32	36	40	20	20	24	22	22	20
8	26	26	25	28	27	31	33	40	20	19	24	22	20	19
9	23	23	22	26	24	28	30	43	17	16	23	20	17	22
10	22	23	21	25	22	27	29	43	19	17	26	21	26	26
11	22	21	21	25	22	25	28	44	16	15	33	18	21	22
12	20	20	20	22	20	24	26	42	18	16	28	20	22	22
13	22	22	21	25	21	27	30	43	18	16	27	22	24	24
14	19	20	19	22	18	25	28	37	16	15	23	34	16	17
15	19	20	19	22	18	25	28	45	17	15	21	37	16	17
16	18	20	18	21	17	24	34	38	17	15	20	33	20	34

TABLE 3.18

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE
6 PHASES, 5 TESTS/PHASE, DETERMINED FAILURE RATE

							LAMBDA SET **********							
PHASE *****	MOD1 ****	MOD2 ****	MOD3 ****	MOD4 ****	MOD5 ****	MOD6 ****	MOD7 ****	MOD8 ****	MOD9 ****	MOD10 *****	MOD11 *****	MOD12 *****	MOD13 *****	MOD14 *****
1	65	65	65	65	65	65	65	65	65	65	65	65	65	65
2	80	74	76	73	79	81	83	76	80	90	67	74	71	78
3	64	67	67	70	70	89	91	81	57	62	76	64	57	56
4	56	58	58	64	61	71	70	69	50	54	59	53	53	50
5	52	53	53	59	52	59	61	83	50	46	68	54	60	58
6	52	53	52	62	52	57	78	88	50	44	55	81	55	77

TABLE 3.19

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

6 PHASES, 10 TESTS/PHASE, DETERMINED FAILURE RATE

							LAMBDA SET **********							
PHASE	MOD1	MOD2	MOD3	MOD4	MOD5	MOD6	MOD7	MOD8	MOD9	MOD10	MOD11	MOD12	MOD13	MOD14
*****	****	****	****	****	****	****	****	****	****	*****	*****	*****	*****	*****
1	46	46	46	46	46	46	46	46	46	46	46	46	46	46
2	56	60	65	64	63	64	63	64	68	54	62	56	61	66
3	54	53	52	55	53	52	57	58	44	50	47	44	49	40
4	46	46	46	51	49	51	53	63	41	41	45	42	40	40
5	39	36	37	46	39	42	50	62	31	30	49	35	35	35
6	42	40	40	49	39	51	57	64	34	32	45	57	42	55

TABLE 3.20

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE
6 PHASES, 20 TESTS/PHASE, DETERMINED FAILURE RATE

						LAMBDA SET **********								
PHASE *****	MOD1 ****	MOD2 ****	MOD3 ****	MOD4 ****	MOD5 ****	MOD6 ****	MOD7 ****	MOD8 ****	MOD9 ****	MOD10 *****	MOD11 *****	MOD12 *****	MOD13 *****	MOD14 *****
1	37	37	37	37	37	51	37	37	37	37	37	37	37	37
2	45	45	44	45	45	37	51	48	45	43	49	44	47	44
3	38	39	39	40	39	38	37	43	33	35	33	33	35	30
4	34	34	33	38	35	35	40	43	27	27	28	27	26	27
5	31	32	30	35	31	36	37	44	25	24	37	28	27	27
6	24	25	23	27	22	28	34	45	19	19	25	36	25	37

TABLE 3.21

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

16 PHASES, 5 TESTS/PHASE, FORECAST FAILURE RATE

	LAMBDA SET **********													
PHASE *****	1 ****	2 ****	3 ****	4 ****	5 ****	6 ****	7 ****	8 ****	9 ****	10 ****	11 ****	12 ****	13 ****	14 ****
3	1000	1000	973	1000	978	997	1000	890	240	90	994	1000	1000	1000
4	133	168	94	612	161	99	99	76	64	54	764	244	281	133
5	213	140	117	673	207	162	168	48	52	44	69	292	339	135
6	82	61	65	153	75	131	170	39	43	36	41	368	167	52
7	54	55	49	67	54	51	44	33	36	30	34	49	45	29
8	45	45	42	59	49	59	53	30	32	28	32	41	31	22
9	44	41	41	58	48	52	47	26	30	26	30	35	22	21
10	45	39	42	62	46	41	47	22	28	25	27	31	19	21
11	44	36	40	71	45	42	51	21	25	23	65	27	18	21
12	36	28	34	45	35	30	38	19	21	20	46	22	16	18
13	33	27	31	41	31	29	43	18	20	19	34	23	15	17
14	30	27	28	36	28	32	47	17	19	18	33	51	15	15
15	29	27	27	37	26	36	55	17	18	17	30	83	19	22
16	28	26	26	36	24	41	144	16	17	16	30	98	28	112

TABLE 3.22

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

16 PHASES, 10 TESTS/PHASE, FORECAST FAILURE RATE

LAMBDA SET
****

PHASE	1	2	3	4	5	6	7	8	9	10	11	12	13	14
*****	****	****	****	****	****	****	****	****	****	****	****	****	****	****
3	739	994	243	1000	997	998	1000	614	89	56	1000	1000	1000	978
4	93	806	76	240	129	615	114	550	52	42	192	111	329	118
5	70	644	58	179	76	78	75	58	43	35	157	76	84	58
6	60	59	54	105	85	64	64	52	40	31	57	46	53	41
7	49	59	46	63	54	57	56	48	37	27	51	40	39	25
8	43	51	39	61	46	49	54	41	30	22	45	34	26	22
9	36	49	33	49	38	53	56	40	27	21	37	32	19	23
10	37	53	34	48	37	56	59	40	27	20	37	33	20	23
11	40	66	35	53	37	80	84	35	26	20	91	33	20	22
12	38	61	34	48	34	70	73	33	24	19	64	33	19	21
13	38	67	33	46	32	78	73	30	24	18	49	40	17	19
14	29	33	27	34	26	39	47	29	23	18	37	93	16	17
15	26	30	25	31	24	38	47	28	22	17	34	108	19	20
16	25	27	23	29	21	36	85	24	19	15	29	63	26	73

TABLE 3.23

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

16 PHASES, 20 TESTS/PHASE, FORECAST FAILURE RATE

	LAMBDA SET **********													
PHASE *****	1 ****	2 ****	3 ****	4 ****	5 ****	6 ****	7 ****	8 ****	9 ****	10 ****	11 ****	12 ****	13 ****	14 ****
3	1000	949	820	874	1000	425	988	991	114	41	908	759	762	1000
4	233	334	111	999	340	437	1000	755	36	26	95	782	656	244
5	54	62	49	78	60	89	79	46	32	24	43	51	59	48
6	42	71	38	58	45	56	64	37	28	21	36	39	41	29
7	33	37	31	39	34	53	52	35	25	19	32	29	27	18
8	26	29	25	31	27	51	64	33	21	17	26	24	19	14
9	27	32	25	32	27	53	80	29	20	16	25	24	16	15
10	25	29	23	31	25	46	57	24	19	15	24	22	13	16
11	22	26	21	26	21	35	40	23	19	14	58	21	15	17
12	22	25	21	27	21	32	39	23	17	13	42	19	13	15
13	20	22	19	23	18	28	33	22	16	13	33	20	11	13
14	20	22	19	24	18	29	35	22	16	12	27	52	11	11
15	19	22	18	22	17	29	35	22	16	12	24	63	14	15
16	18	21	17	21	15	29	61	22	15	11	20	51	20	49

TABLE 3.24

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

6 PHASES, 5 TESTS/PHASE, FORECAST FAILURE RATE

	LAMBDA SET **********													
PHASE *****	MOD1 ****	MOD2 ****	MOD3 ****	MOD4 ****	MOD5 ****	MOD6 ****	MOD7 ****	MOD8 ****	MOD9 ****	MOD10 *****	MOD11 *****	MOD12 *****	MOD13 *****	MOD14 *****
3	212	626	138	897	223	962	1000	888	174	86	1000	934	442	74
4	172	116	108	745	146	132	999	81	69	57	325	116	37	43
5	62	87	60	81	61	123	133	53	51	47	475	77	34	32
6	55	83	51	71	46	147	273	38	42	36	208	857	77	403

TABLE 3.25

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

6 PHASES, 10 TESTS/PHASE, FORECAST FAILURE RATE

LAMBDA SET

PHASE	MOD1	MOD2	MOD3	MOD4	MOD5	MOD6	MOD7	MOD8	MOD9	MOD10	MOD11	MOD12	MOD13	MOD14
3	87	276	84	195	110	1000	996	1000	73	50	512	84	85	58
4	67	121	59	103	65	589	647	161	49	44	443	71	34	31
5	51	60	46	74	49	134	567	61	38	34	240	47	28	29
6	47	59	42	67	40	118	300	44	32	26	92	454	59	164

TABLE 3.26

INSTANTANEOUS RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

6 PHASES, 20 TESTS/PHASE, FORECAST FAILURE RATE

								LAMBDA SET						
PHASE	MOD1	MOD2	MOD3	MOD4	MOD5	MOD6	MOD7	MOD8	MOD9	MOD10	MOD11	MOD12	MOD13	MOD14
3	76	268	63	245	95	916	658	410	51	39	198	67	69	36
4	50	87	45	77	50	831	999	117	36	31	132	47	23	23
5	46	85	40	67	41	473	971	77	33	25	991	67	19	20
6	38	62	34	51	31	70	180	37	29	22	42	1000	40	95

The modification number m takes on values 0, 1, 2, . . . with m = 0 designating the original version of the component. Component reliability R_m may be thought of as an instantaneous or modification reliability as opposed to a cumulative or average reliability since it is the intrinsic reliability of the m^{th} modification version of the item rather than a characteristic reliability for all m+1 versions of the item that have been produced. Under the assumption of reliability growth, the quantity exp $(\alpha + \beta m)$ in equation 3.56 is expected to increase as modifications to the component are made which hopefully improve its reliability. Hence, as modifications are accomplished the fraction 1/exp $(\alpha + \beta m)$ in equation 3.56 decreases and component reliability increases toward the maximum reliability of 1.0. The model is considered discrete since the single independent or control variable m, modification number, is discrete as opposed to a continuous variable such as total accumulated test time.

A mathematical analysis of the discrete reliability growth model is given in reference 1. Other Monte Carlo simulation evaluations and comparisons of the discrete model are contained in references 5 and 9.

2. Reliability Testing Procedure

For the discrete reliability growth model the appropriate reliability testing procedure of a system acquisition cycle is one in which modifications to an item are made as a specified number of failures of the item are observed during the testing procedure. The modifications of the item are made to improve reliability and hopefully result in such improvement. Modifications are indexed by m with a total of NTM modifications being considered during an acquisition cycle; i.e., m = 0, 1, 2, 3, ... , NTM. The total number of item failures that are observed during testing of the m^{th} modification version of an item before the item

is modified again is denoted by NTF_m . The total number of item failures NTF_m is specified before reliability testing is conducted and may be varied from modification to modification. During testing of each modification version of an item, an underlying, inherent reliability that is unknown to the project managers/contractors is present in the item. The underlying modification reliability of the m^{th} modification version of the component is denoted as R_m .

Between each failure of the m^{th} modification version of an item a number of tests of the item are conducted which are denoted as $NT_{m,j}$; i.e., the number of tests run on the m^{th} modification version of the item between the $(j-1)^{th}$ failure and the j^{th} failure. $NT_{m,j}$ includes the j^{th} failure. So, $NT_{m,1}$ is the number of tests performed thru the first observed failure of the item when the underlying modification reliability is R_m . $NT_{m,2}$ is the number of tests performed from the first failure thru the second failure of the item while the underlying modification reliability remains R_m and so on. After NTF_m failures of the item under test are observed, a modification is accomplished and the underlying modification reliability of the component changes to R_{m+1} . Therefore, the failure number index j runs from 1 to NTF_m for each modification m ; i.e., $NT_{m,1}$, $NT_{m,2}$, ... , NT_{m,NTF_m} . All $NT_{m,j}$ are random variables and cannot be specified beforehand. This is quite different from the testing procedure for the continuous reliability growth models evaluated where the number of tests per test phase were specified prior to any testing being performed.

Figure 3.78 is a schematic diagram of the discrete model reliability testing procedure. The total number of modifications NTM and the total number of failures permitted before modification NTF_m are

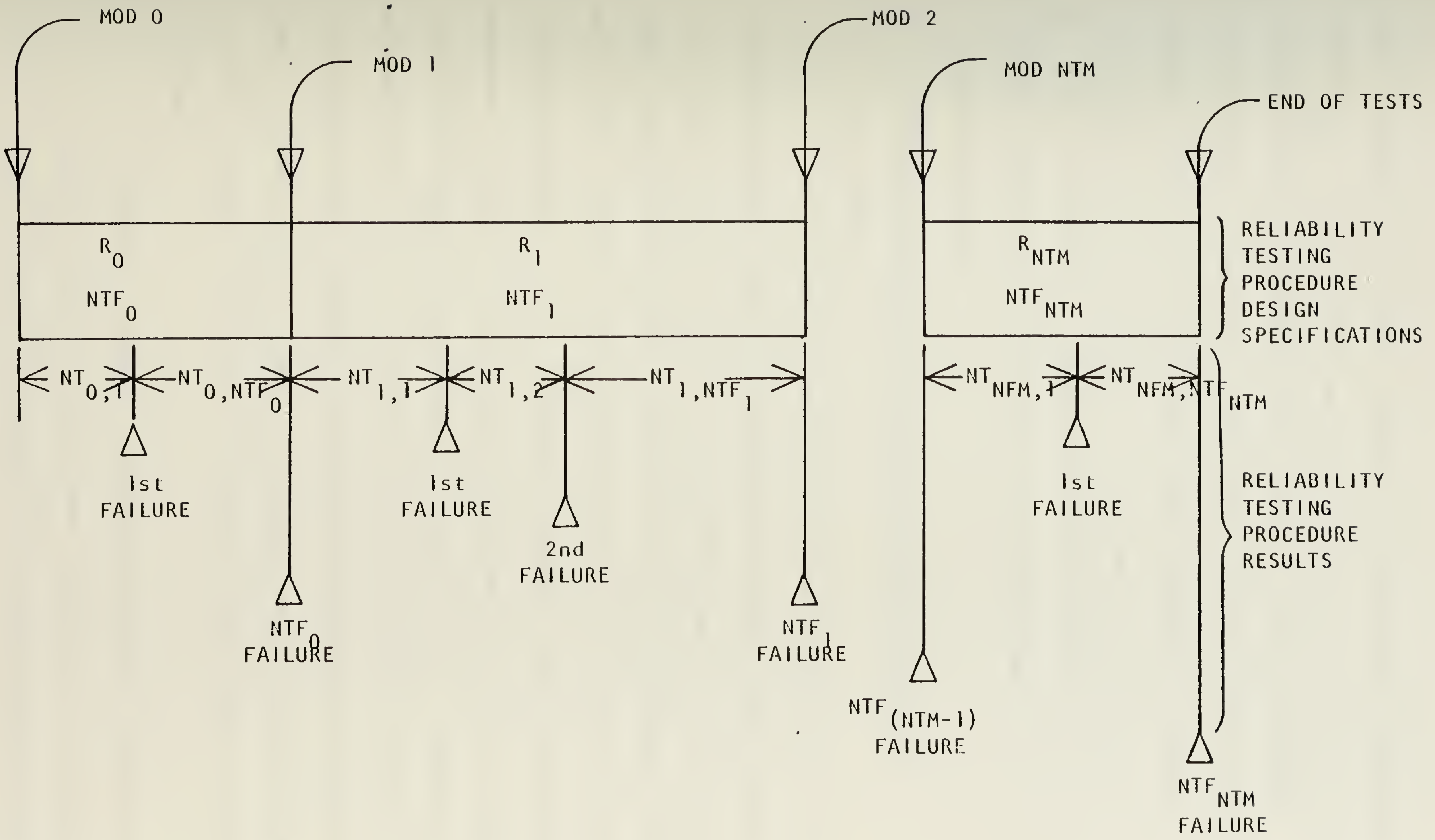

FIGURE 3.78

DISCRETE RELIABILITY GROWTH MODEL RELIABILITY TESTING PROCEDURE DIAGRAM

specified prior to testing. The number of tests performed between failures $NT_{m,j}$ for each modification version of the item under test constitute the data collected from the reliability testing procedure. As with the continuous reliability growth models the most salient feature of the computer simulation of the discrete model reliability testing procedure is that the analyst also specifies the underlying modification reliability R_m that is effective for each modification version of the item. Thus an accurate assessment of the accuracy of the model can be made because the R_m values are known.

3. Reliability Testing Procedure Computer Simulation

a. Summary

The reliability testing procedure appropriate to the discrete reliability growth model was simulated on the Naval Postgraduate School W. R. Church Computer Center's IBM 360/67 System utilizing standard computer simulation techniques. Reliability testing procedure design parameters NTM and NTF_m were specified along with various underlying modification reliability sets. Tests of components were then simulated with component success or failure being determined for each test based on the appropriate underlying modification reliability R_m. Number of tests between failures $NT_{m,j}$ data were collected as each procedure simulation was accomplished. Simulations of reliability testing procedures for each specified modification reliability set were replicated one-hundred times in order to assess the discrete reliability growth model's variability performance.

After the reliability testing procedure simulations were completed, discrete reliability growth model estimates of the underlying modification reliability of the component were produced based on the simulated

test data by computations utilizing ordinary least squares regression techniques to estimate the model parameters α and β. Mean and variability statistics of the model reliability estimates were then computed to permit performance evaluation of the discrete reliability growth model.

b. Detail Description

The computer simulation of the reliability testing procedure appropriate to the discrete reliability growth model was initiated by reading in the following reliability testing procedure design specifications: NTM, the total number of modifications to be made for the acquisition cycle under consideration; NTF_m, the total number of failures of the m^{th} modification version of the item to be observed until the $(m+1)^{th}$ modification is made; R_m, the underlying, intrinsic modification reliability of the m^{th} modification version of the item; and NSIMS, the number of times the specified reliability testing procedure was to be simulated. For this thesis NSIMS was always specified as one-hundred simulations.

After specification data were read into the computer, uniform (0,1) random variates were drawn in sequence corresponding to component tests starting for the modification 0 type component; i.e., uniform random variates were drawn in sequence and indexed as $U_{m,j,i}$ for m = 0, 1, 2, ..., NTM; j = 1, 2, 3, ... , NTF_m; and i = 1, 2, 3, ... starting with the $U_{0,1,1}$ test.

Each uniform variate was then compared with the appropriate underlying modification reliability R_m to determine if a success or a failure occurred during the test. The number of tests $NT_{m,j}$ and the number of failures accumulated NF_m data were recorded after each simulated test such that

$$NT_{m,j} = NT_{m,j} + 1 \text{ (}NT_{m,j}\text{ incremented by one test), and} \qquad (3.57)$$

$$NF_m = \begin{cases} NF_m \text{ if } U_{m,j,i} < R_m & \text{(success)} \\ NF_m + 1 \text{ if } U_{m,j,i} \geq R_m & \text{(failure)} \end{cases} \qquad (3.58)$$

for $m = 0, 1, 2, \ldots, NTM$; $j = 1, 2, 3, \ldots, NTF_m$; and $i = 1, 2, 3, \ldots$ If the test was a success, then the simulation proceeded to the next test. If the test was a failure, then the number of failures accumulated NF_m was compared to the total number of failures until modification NTF_m to determine if a modification was appropriate. If NF_m was less than NTF_m, testing continued for the given modification and number of tests were then counted until the next failure; i.e., $NT_{m,j+1}$. If NF_m was equal to NTF_m, then a modification of the item under test was simulated and testing continued for the next modification; i.e., NF_{m+1} and $NT_{m+1,1}$ being tallied next.

Testing in this sequence was simulated until NF_{NTM} was equal to NTF_{NTM} which signalled the termination of a given reliability testing procedure computer simulation. The simulation resulted in the number of tests until a failure $NT_{m,j}$ data being recorded for $m = 0, 1, 2, \ldots, NTM$ and $j = 1, 2, 3, \ldots, NTF_m$. These data constituted the total data gathered from a single computer simulation of the discrete model reliability testing procedure for a single specified underlying modification reliability progress path; i.e., a specified set of reliabilities which shall henceforth be referred to as a reliability set. The reliability testing procedure was then replicated one-hundred times ($r = 1, 2, 3, \ldots, NSIMS = 100$) such that reliability testing procedure results data were actually indexed as $NT_{m,j,r}$. Since ten reliability sets were utilized

for evaluating the discrete reliability growth model with three different total number of failures until modification NTF_m specified for each reliability set, 3000 reliability testing procedures were simulated for the discrete reliability growth model evaluation.

4. Computer Simulation Data Manipulation

The discrete reliability growth model of equation 3.56 is proposed as a model of the unknown, underlying modification reliability R_m intrinsic to a component as it proceeds thru a system acquisition cycle. It is against this true reliability R_m that the model's performance was measured.

Equation 3.56 may be given in the form

$$R_m = 1 - \exp(-\alpha - \beta m) \quad \text{or} \tag{3.59}$$

$$\exp(-\alpha - \beta m) = 1 - R_m \tag{3.60}$$

for m = 0, 1, 2, ... , NTM. The logarithmic transformation of equation 3.60 yields

$$(\alpha + \beta m)_j = -\ln(1 - R_m)_j \tag{3.61}$$

for m = 0, 1, 2, ... , NTM and j = 1, 2, 3, ... , NTF_m .

An unbiased estimator of the quantity $\alpha + \beta m$ is given in reference 1 as

$$\widehat{(\alpha + \beta m)}_{m,j} = \begin{cases} 0.0 \text{ if } NT_{m,j} = 1 \text{ (first test was failure)} \\ 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{NT_{m,j} - 1} \text{ if } NT_{m,j} \geqq 2 \end{cases} \tag{3.62}$$

for m = 0, 1, 2, ... , NTM and j = 1, 2, 3, ... , NTF_m .

By equation 3.61

$$\widehat{-\ln(1-R_m)}_{m,j} = \widehat{(\alpha + \beta m)}_{m,j} \tag{3.63}$$

for $m = 0, 1, 2, \ldots, NTM$ and $j = 1, 2, 3, \ldots, NTF_m$. From this relation an average estimate may be computed as

$$\overline{\widehat{-\ln(1-R_m)}}_m = \overline{\widehat{(\alpha + \beta m)}}_m = \frac{1}{NTF_m} \sum_{j=1}^{NTF_m} \widehat{(\alpha + \beta m)}_{m,j} \ . \tag{3.64}$$

Letting

$$Y_m = \overline{\widehat{-\ln(1-R_m)}}_m ,$$

then equation 3.61 may be written as

$$Y_m = \hat{\alpha}_m + \hat{\beta}_m m \tag{3.65}$$

for $m = 0, 1, 2, \ldots, NTM$ which is of the form $Y = \alpha + \beta X$. Applying ordinary least squares regression estimates as given in reference 6, the α_m and β_m estimates are

$$\hat{\beta}_m = \frac{\sum_{i=0}^{m} (i - \bar{m})\, Y_i}{\sum_{i=0}^{m} (i - \bar{m})^2} \quad \text{and} \tag{3.66}$$

$$\hat{\alpha}_m = \bar{Y} - \hat{\beta}_m \bar{m} \tag{3.67}$$

for m = 1, 2, 3, ... , NTM where

$$\bar{Y} = \frac{1}{m+1} \sum_{i=0}^{m} Y_i \quad \text{and} \tag{3.68}$$

$$\bar{m} = \frac{1}{m+1} \sum_{i=0}^{m} i \ . \tag{3.69}$$

Finally, these $\widehat{\alpha}_m$ and $\widehat{\beta}_m$ estimates are utilized in the discrete reliability model, equation 3.56, to produce the model estimates of the unknown, underlying modification reliability. These estimates are given by

$$\widehat{R}_m = 1 - \frac{1}{\exp\ (\widehat{\alpha}_m + \widehat{\beta}_m\ m)} \tag{3.70}$$

for m = 1, 2, 3, ... , NTM. Note than since the regression procedure requires a minimum of two observations, model reliability estimates are produced from the first modification thru the last modification (NTM). A reliability estimate for original version of the component is derived from equation 3.56 and 3.64 as

$$\widehat{R}_0 = 1 - \frac{1}{\exp\ \widehat{(\alpha + \beta m)}_0} \ . \tag{3.71}$$

Again, since the reliability testing procedure was simulated one-hundred times (r = 1, 2, 3, ... , NSIMS = 100) for each reliability set, one-hundred estimates of the modification reliability were obtained for each modification version of the component under test. The mean value, standard deviation, and percentage standard error of these estimates

were computed as

$$\overline{\widehat{R}}_m = \frac{1}{NSIMS} \sum_{r=1}^{NSIMS} \widehat{R}_{m,r} \ , \qquad (3.72)$$

$$S.D._{\widehat{R}_m} = \sqrt{\frac{1}{NSIMS-1} \sum_{r=1}^{NSIMS} (\widehat{R}_{m,r} - \overline{\widehat{R}}_m)^2} \ , \text{ and} \qquad (3.73)$$

$$P.S.E._{\widehat{R}_m} = \frac{S.D._{\widehat{R}_m}}{\overline{\widehat{R}}_m} \times 100 \qquad (3.74)$$

for m = 0, 1, 2, ... , NTM.

To examine the discrete reliability growth model's forecasting capability a next modification reliability forecast was made for modifications m = 2, 3, 4, ... , NTM. Forecasting started for the second modification since model parameter estimates were not available until testing under the first modification was completed. No forecast was made past modification NTM because there would be no underlying modification reliability with which to compare. Forecasts were made as

$$\widehat{FR}_{m+1} = 1 - \frac{1}{\exp\ (\widehat{\alpha}_m + \widehat{\beta}_m\ (m+1))} \qquad (3.75)$$

for m = 2, 3, 4, ... , NTM. Finally, forecast modification reliability statistics similar to the determined modification reliability statistics in equations 3.72, 3.73, and 3.74 were computed as

$$\overline{\widehat{FR}}_m = \frac{1}{NSIMS} \sum_{r=1}^{NSIMS} \widehat{FR}_{m,r} \ , \qquad (3.76)$$

$$S.D._{\widehat{FR}_m} = \sqrt{\frac{1}{NSIMS-1} \sum_{r=1}^{NSIMS} (\widehat{FR}_{m,r} - \overline{\widehat{FR}}_m)^2} \text{ , and} \tag{3.77}$$

$$P.S.E._{\widehat{FR}_m} = \frac{S.D._{\widehat{FR}_m}}{\overline{\widehat{FR}}_m} \times 100 \tag{3.78}$$

for m = 2, 3, 4, ... , NTM.

To evaluate the discrete reliability growth model ten reliability sets were formed for use as the specified underlying reliability progress paths. These reliability sets are listed in table 3.27. All ten reliability progress paths start at a relatively low reliability of 0.200 for the original version of the component under scrutiny; i.e., $R_0 = 0.200$ for reliability sets 1 thru 10. The paths display reliability progress that ranges from extremely rapid reliability growth (reliability set 1; $R_0 = 0.200$, $R_1 = 0.925$) to linear growth (reliability set 6; $R_{m+1} = R_m + 0.150$) to permanently stagnated reliability progress (reliability set 10; R_0 thru $R_6 = 0.200$). Reliability progress was simulated only for components that were modified a total of five times during the acquisition cycle; i.e., NTM = 5 and m = 0, 1, 2, 3, 4, 5. Each reliability set progress path was simulated for three different total number of failures until modification specifications; i.e., NTF_m = 1, 3, and 5 failures which were held constant from modification to modification.

5. Model Performance

a. Accuracy Performance

Figures 3.79 thru 3.98 present all the cases (ten reliability sets) of the discrete reliability growth model's capability to determine and forecast the unknown, underlying modification reliability progress

TABLE 3.27

RELIABILITY SETS
DISCRETE RELIABILITY GROWTH MODEL

	RELIABILITY SET									
MODIFICATION	1	2	3	4	5	6	7	8	9	10
************	****	****	****	****	****	****	****	****	****	****
0	.200	.200	.200	.200	.200	.200	.200	.200	.200	.200
1	.925	.850	.700	.550	.450	.350	.225	.500	.650	.200
2	.950	.900	.825	.750	.650	.500	.275	.550	.650	.200
3	.950	.925	.900	.875	.775	.650	.350	.550	.350	.200
4	.950	.945	.925	.925	.875	.800	.475	.700	.600	.200
5	.950	.950	.950	.950	.950	.950	.950	.950	.950	.200

path during a reliability testing procedure for total number of failures until modification $NTF_m = 1$ and 5. These graphs display the "least data" and "most data" cases which can be contrasted for change and improvement as the testing sample is increased. For the $NTF_m = 3$ cases, the accuracy performance fell uniformly between the $NTF_m = 1$ and $NTF_m = 5$ accuracy performance.

The graphs depict the specified true underlying modification reliability R_m progress path (——, solid line), the mean model determined modification reliability $\overline{\widehat{R}_m}$ from equation 3.72 for each modification version of the component under test (⊙ , circles), and the mean model forecast modification reliability $\overline{\widehat{FR}_m}$ from equation 3.76 for the second thru the final modification version of the component (X , crosses) plotted versus the modification number m = 0, 1, 2, 3, 4, 5. Note that the point plotted for the mean model determined modification reliability of the original version (m = 0) of the component under test is not model determined; rather, it is the mean value of the estimate given by the estimator of equation 3.71. This point allows the accuracy of the reliability estimator utilized to be examined also.

To illustrate those quantities graphed in the accuracy performance figures, observe on figure 3.83 that for the third modification, the specified underlying reliability R_3 was 0.90, the mean model determined modification reliability $\overline{\widehat{R}_3}$ was approximately 0.86, and the mean model forecast modification reliability $\overline{\widehat{FR}_3}$ made from the second modification was approximately 0.77. Since this graph is for reliability set 3, referring to table 3.27, the specified modification reliability for the third modification of reliability set 3 is 0.900 as is graphed on figure 3.83. Finally, note on figure 3.83 for the unmodified version

of the component tested (m = 0) that the reliability estimator of equation 3.71 produced a mean estimate $\overline{\widehat{R}_0}$ of approximately 0.175 for the true underlying reliability R_o of 0.200.

With the exception of reliability set 9 (figures 3.95 and 3.96) the discrete model determines the shape of the underlying reliability progress path with very good accuracy. Forecasting accuracy, while fairly good for the "nice" reliability progress path sets 1 thru 4, displays difficulty with the anomalous reliability progress paths characterized in reliability sets 6 thru 10. The discrete model occasionally produced negative mean forecast estimates of reliability. The negative mean forecast reliability estimates, the magnitude of which never exceeded 0.20, are plotted as 0.0 on the accuracy performance graphs. For example see figure 3.97 for mean forecasts at modifications 2, 3, and 4. No negative mean forecasts of reliability were produced when the total number of failures until modification was specified as five ($NTF_m = 5$); i.e., the "most data" case.

The discrete model accuracy performance graphs display the marked improvement that takes place both in determining and forecasting reliability status as the specified total number of failures until modification NTF_m is increased from one to five which essentially increases the test data which the model can utilize. The improvement in accuracy is also displayed in the $NTF_m = 3$ accuracy performance graphs which are not presented. The reliability estimator of equation 3.71 displays the same improvement in accuracy when the testing sample size is increased as may be seen by examining the graph pairs for $NTF_m = 1$ and $NTF_m = 5$ at the modification m = 0 point.

Figures 3.79 thru 3.98 reveal that the discrete reliability model's accuracy performance for determining the underlying reliability

is in general slightly pessimistic in the instances when mean estimates are not "on the money" as in figures 3.84 and 3.86. Except for the reliability sets 1 and 2, $NTF_m = 5$ cases shown in figures 3.80 and 3.82, mean model forecasting accuracy tends to run from slightly pessimistic as in figure 3.90; reliability set 6, $NTF_m = 5$ to grossly pessimistic as in figure 3.91; reliability set 7, $NTF_m = 1$ or figure 3.97; reliability set 10, $NTF_m = 1$. The general trend evidenced by the graphs is that the less test data available ($NTF_m = 1$) the more pessimistic the mean model estimates, determined or forecast. The same is true of the reliability estimator of equation 3.71.

Reliability set 9 which characterizes a reliability growth pattern interrupted by a period of reliability degradation is the only set with which the discrete model experiences a significant problem. While not detecting the full magnitude of the degradation, the model's mean determined reliability estimates do display the period of degradation credibly. See figures 3.95 and 3.96. However, the model's mean forecast modification reliability estimates are quite contrary to the true underlying reliability progress path. The significant problem being that the period of degradation is not forecast until the true underlying reliability status has recovered from the period of degradation and is in fact displaying significant reliability growth. This behavior is apparent even for the "most data" case $NTF_m = 5$ in figure 3.96. While model forecasting capability suffers in the "least data" case for reliability set 10, $NTF_m = 1$, the discrete model copes with permanent reliability stagnation satisfactorily as seen in figures 3.97 and 3.98.

b. Variability (Precision) Performance

Tables 3.28 thru 3.33 present the discrete reliability growth model's variability performance for determining and forecasting

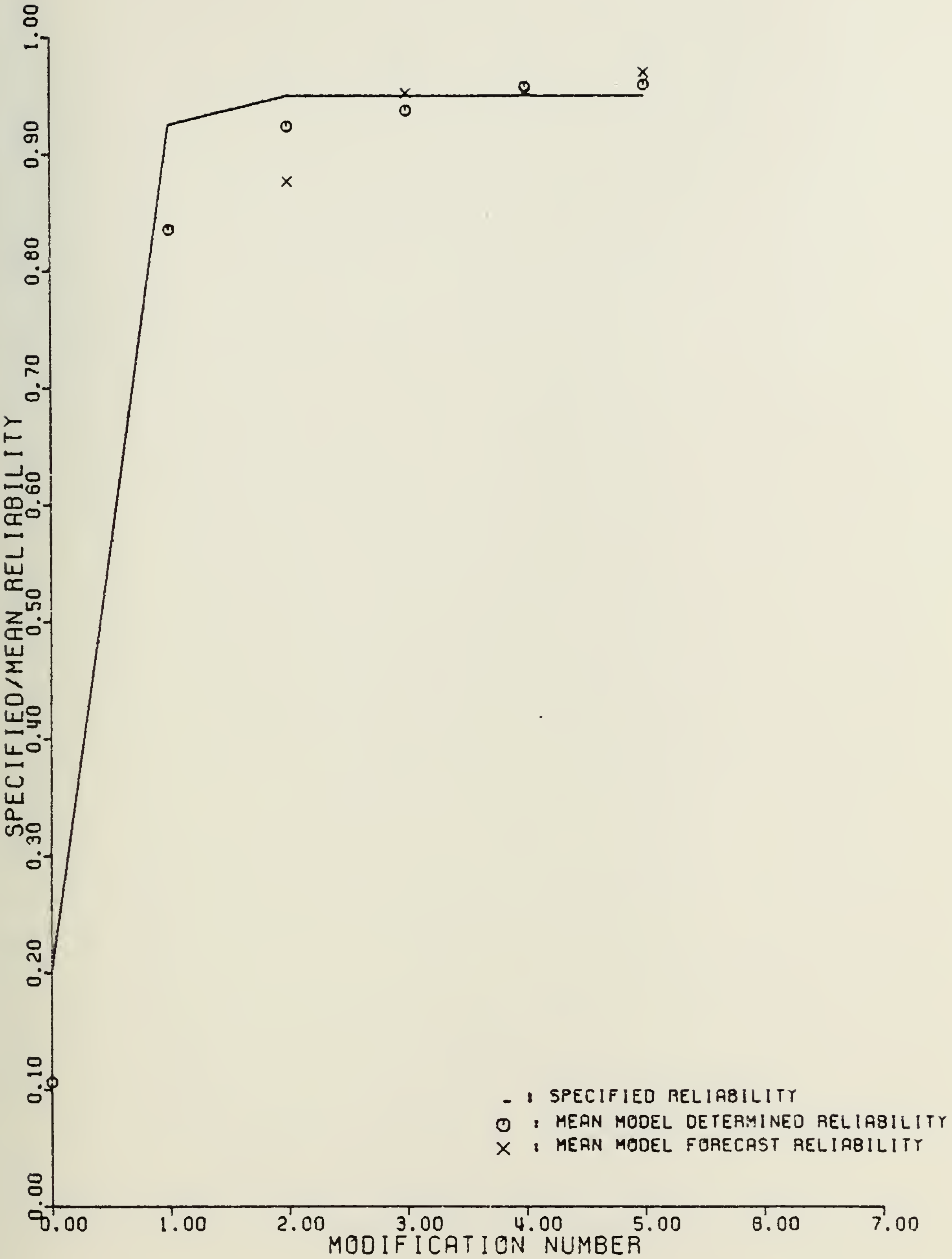

FIGURE 3.79

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 1: 5 MODS, 1 FAILURE/MOD

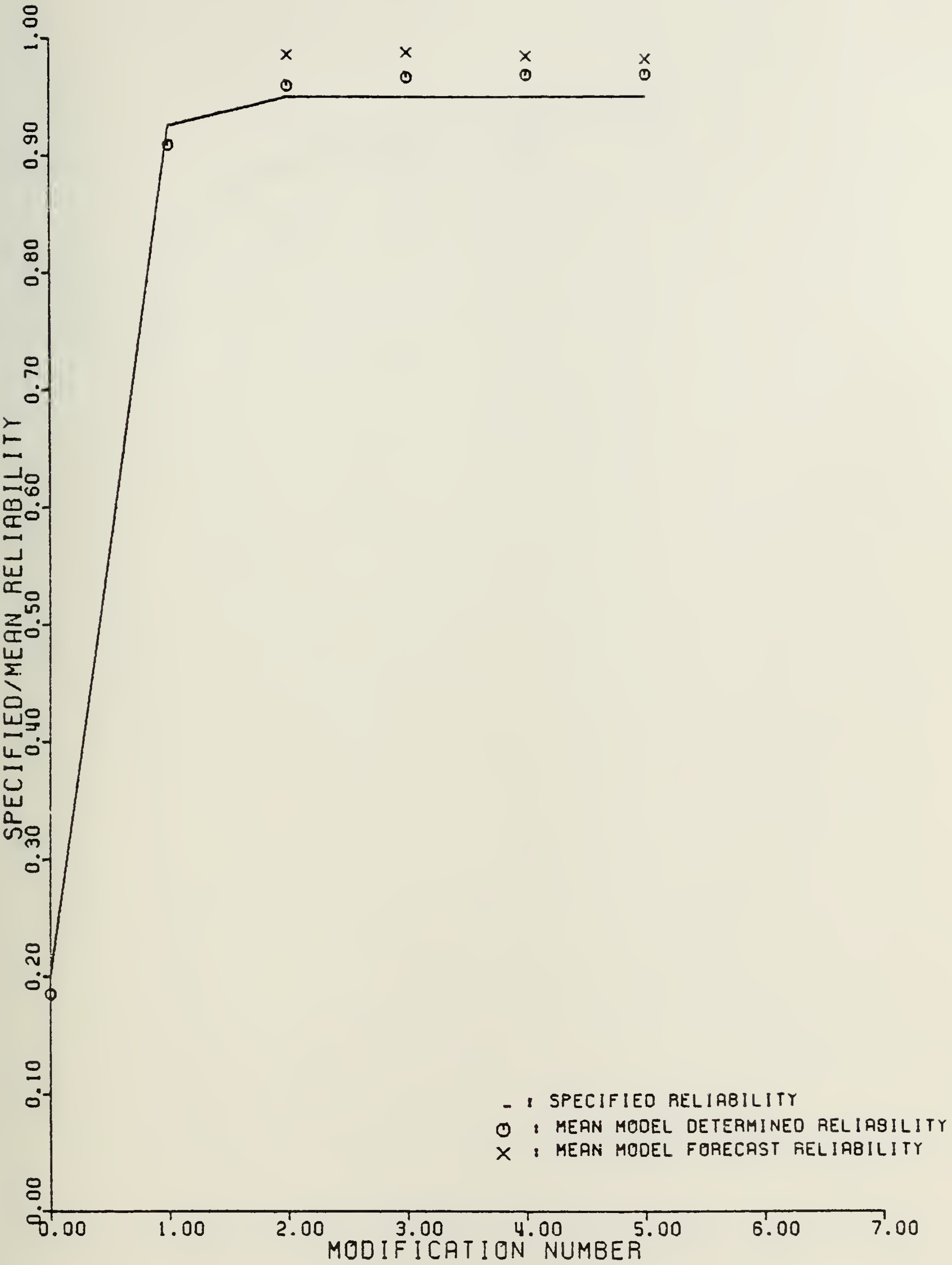

FIGURE 3.80

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 1: 5 MODS, 5 FAILURES/MOD

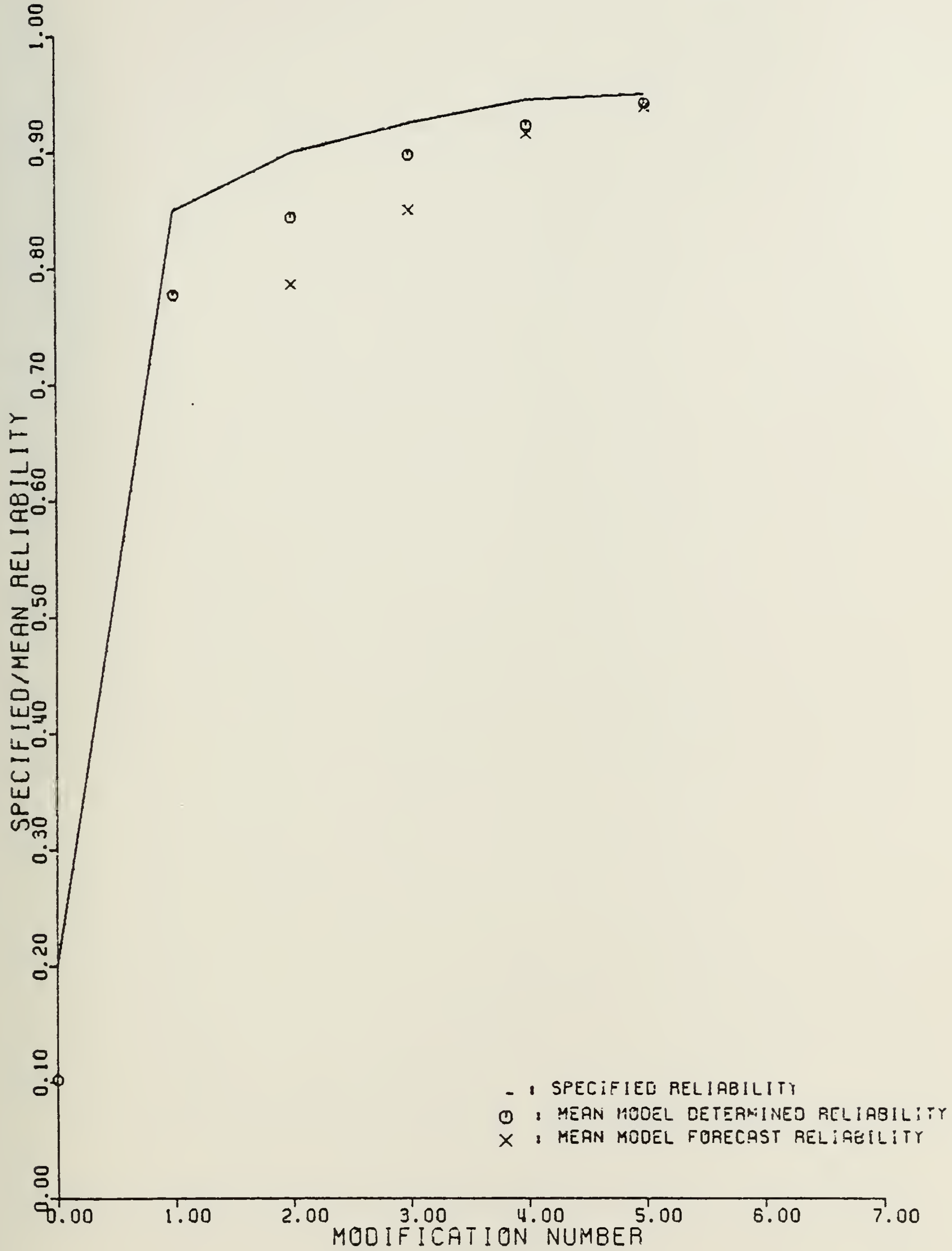

FIGURE 3.81

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 2: 5 MODS, 1 FAILURE/MOD

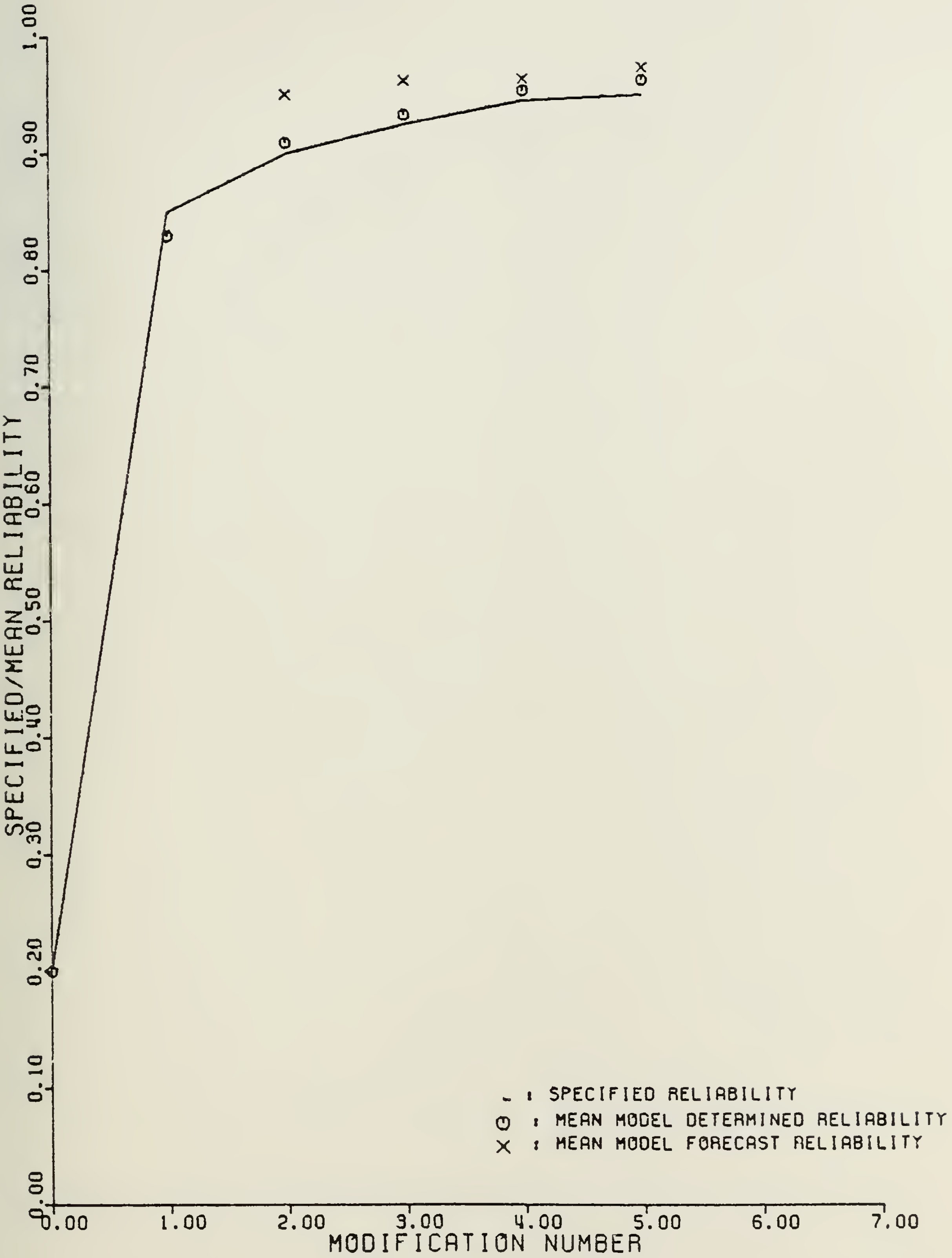

FIGURE 3.82

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 2: 5 MODS, 5 FAILURES/MOD

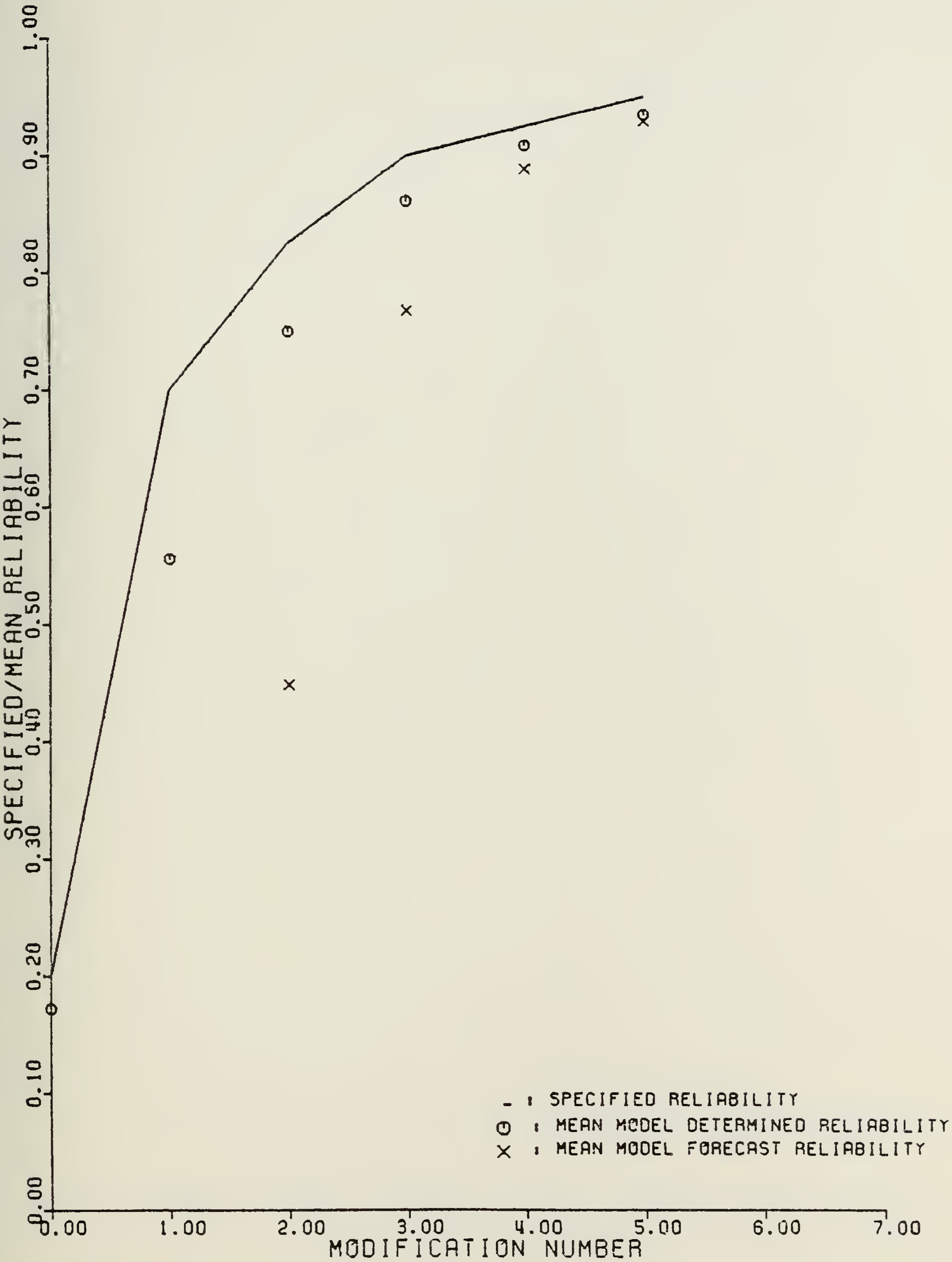

FIGURE 3.83

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 3: 5 MODS, 1 FAILURE/MOD

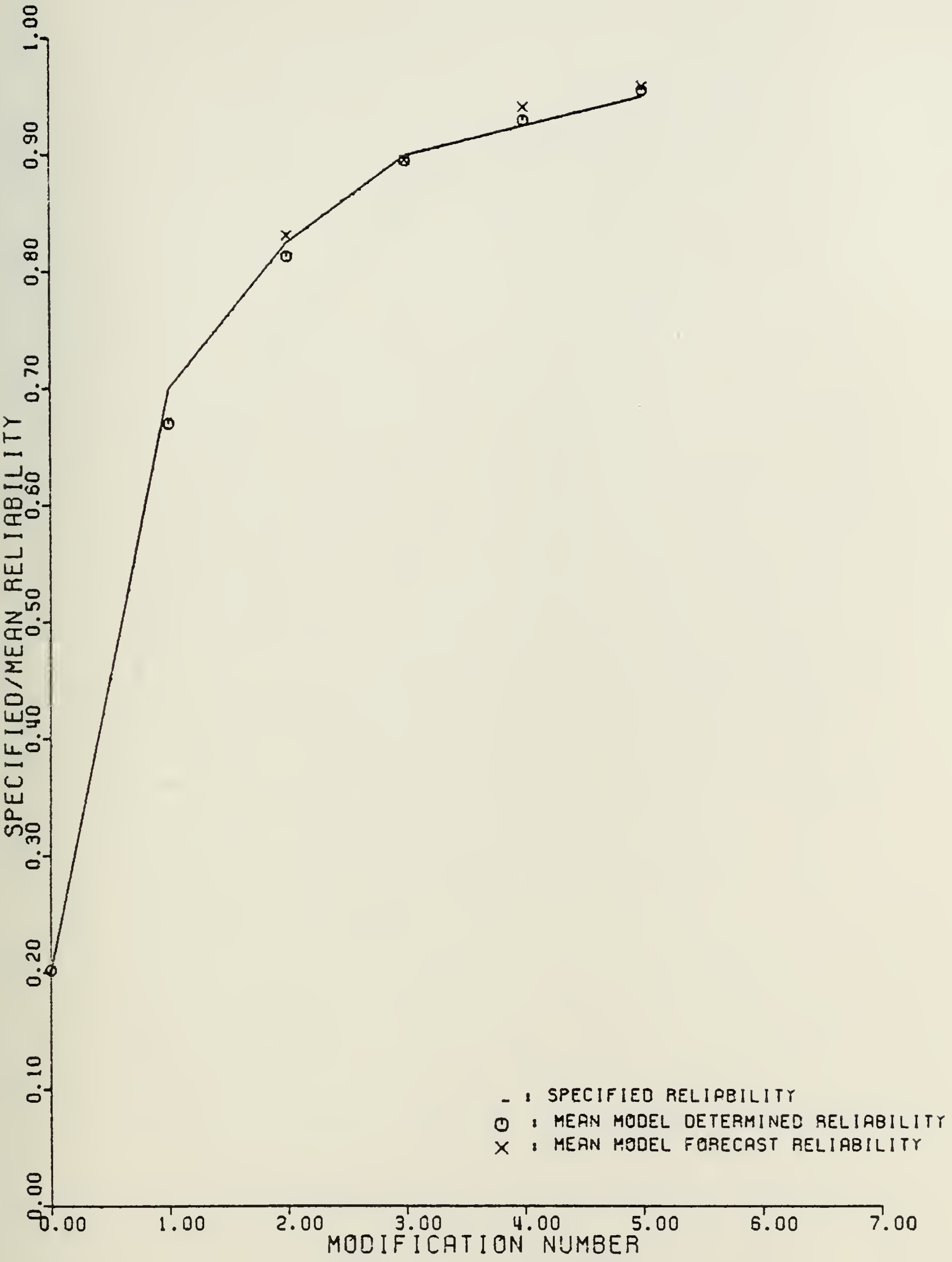

FIGURE 3.84

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 3: 5 MODS, 5 FAILURES/MOD

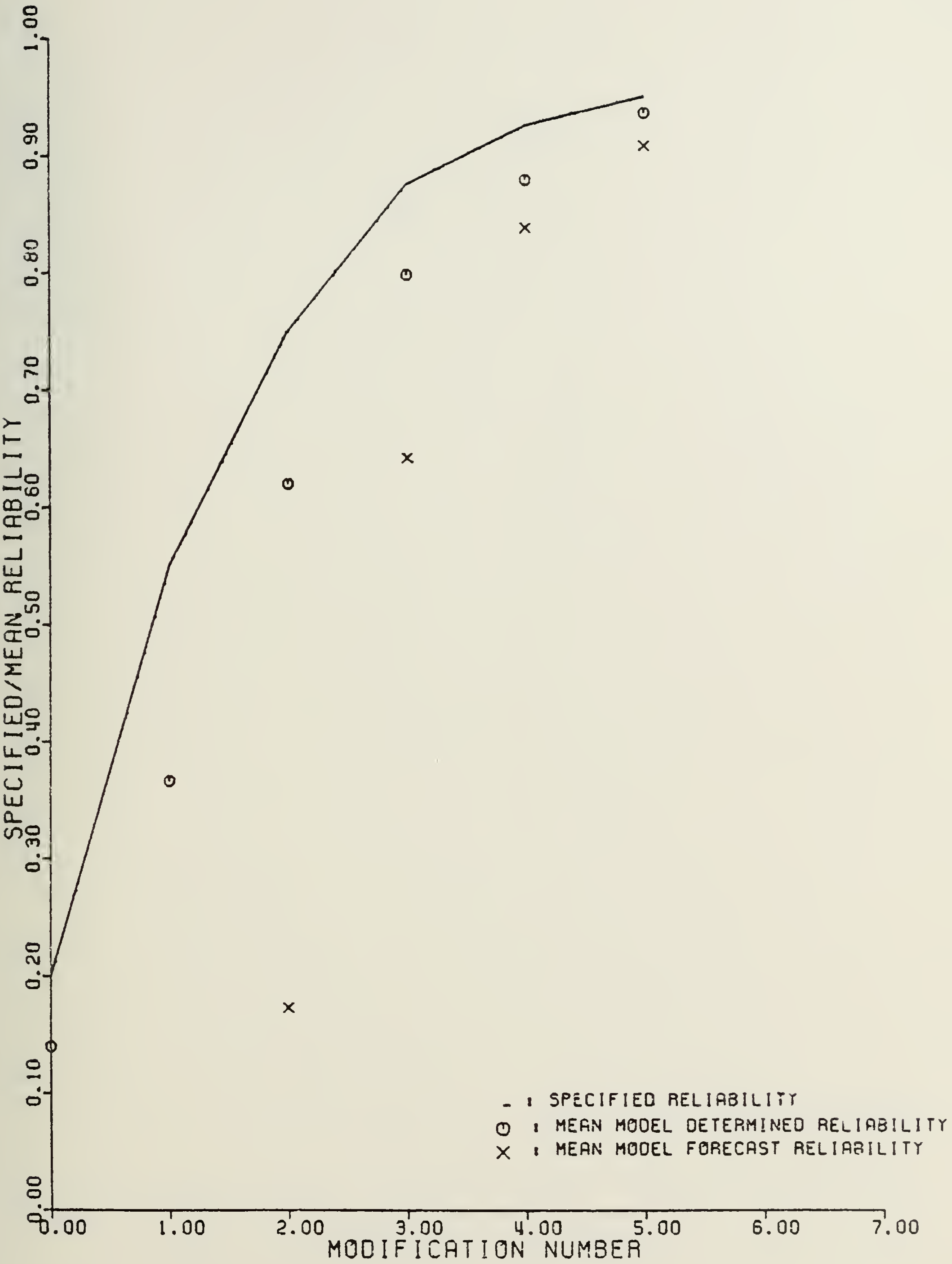

FIGURE 3.85

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 4: 5 MODS, 1 FAILURE/MOD

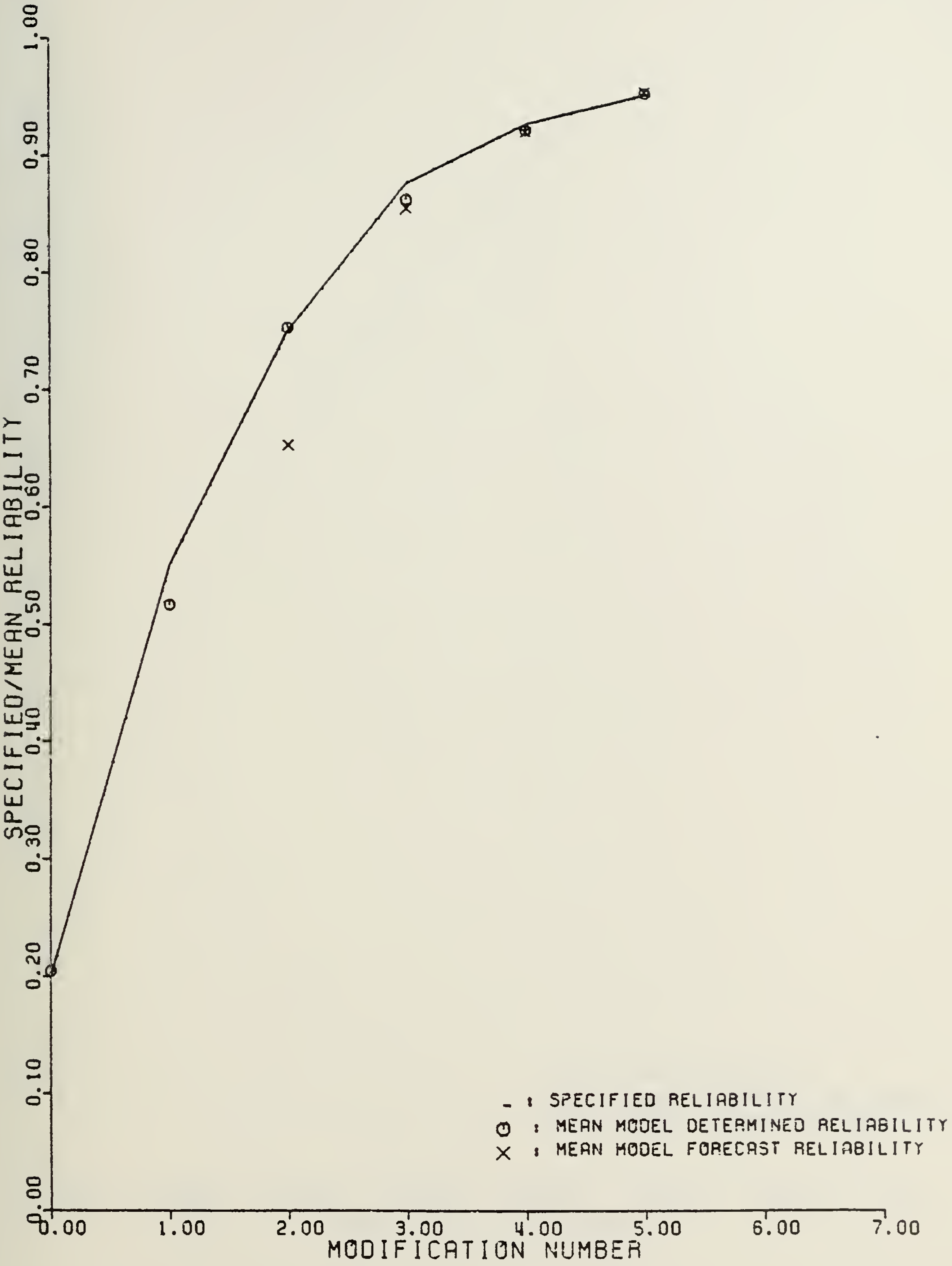

FIGURE 3.86

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 4: 5 MODS, 5 FAILURES/MOD

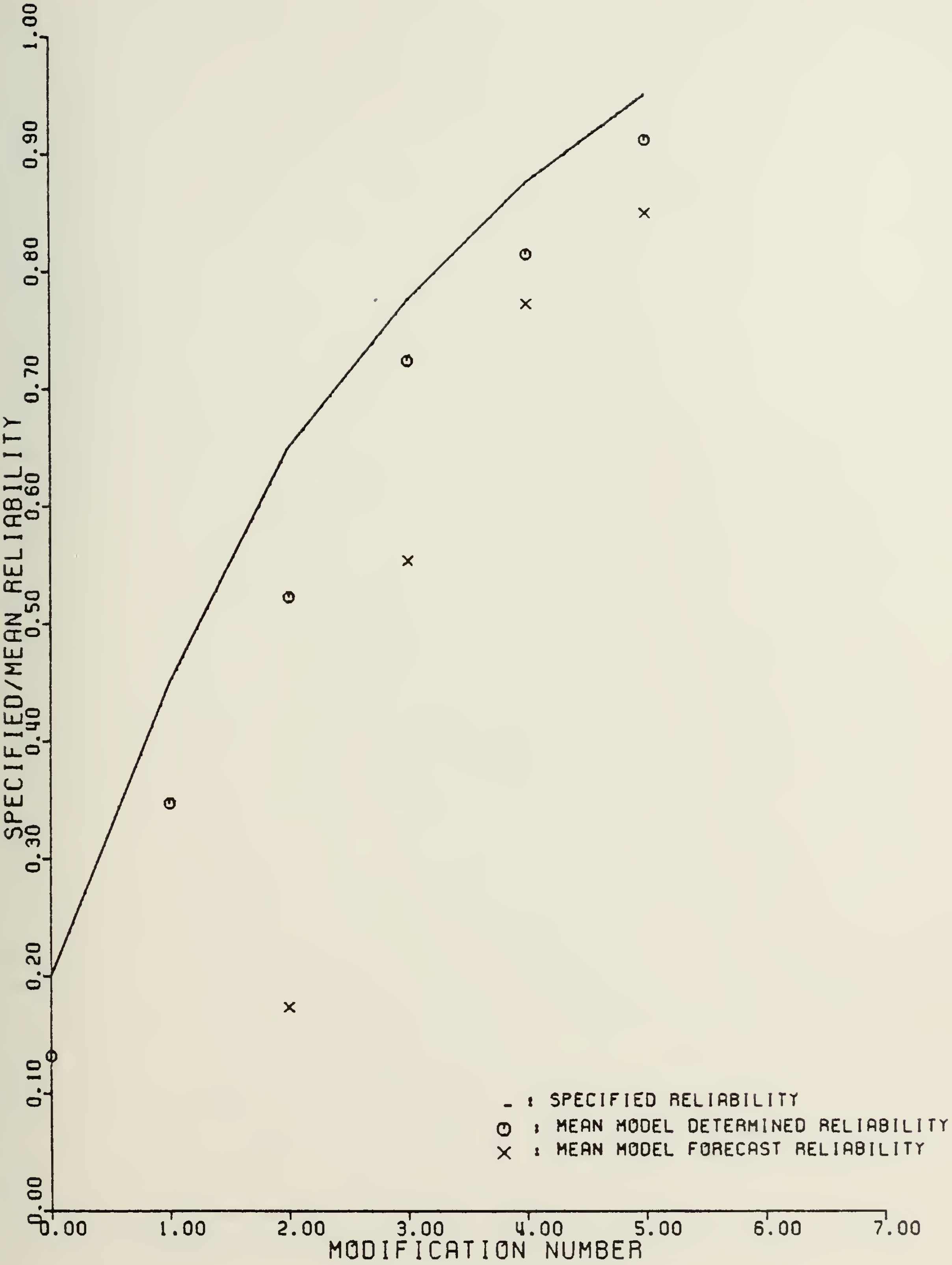

FIGURE 3.87

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 5: 5 MODS, 1 FAILURE/MOD

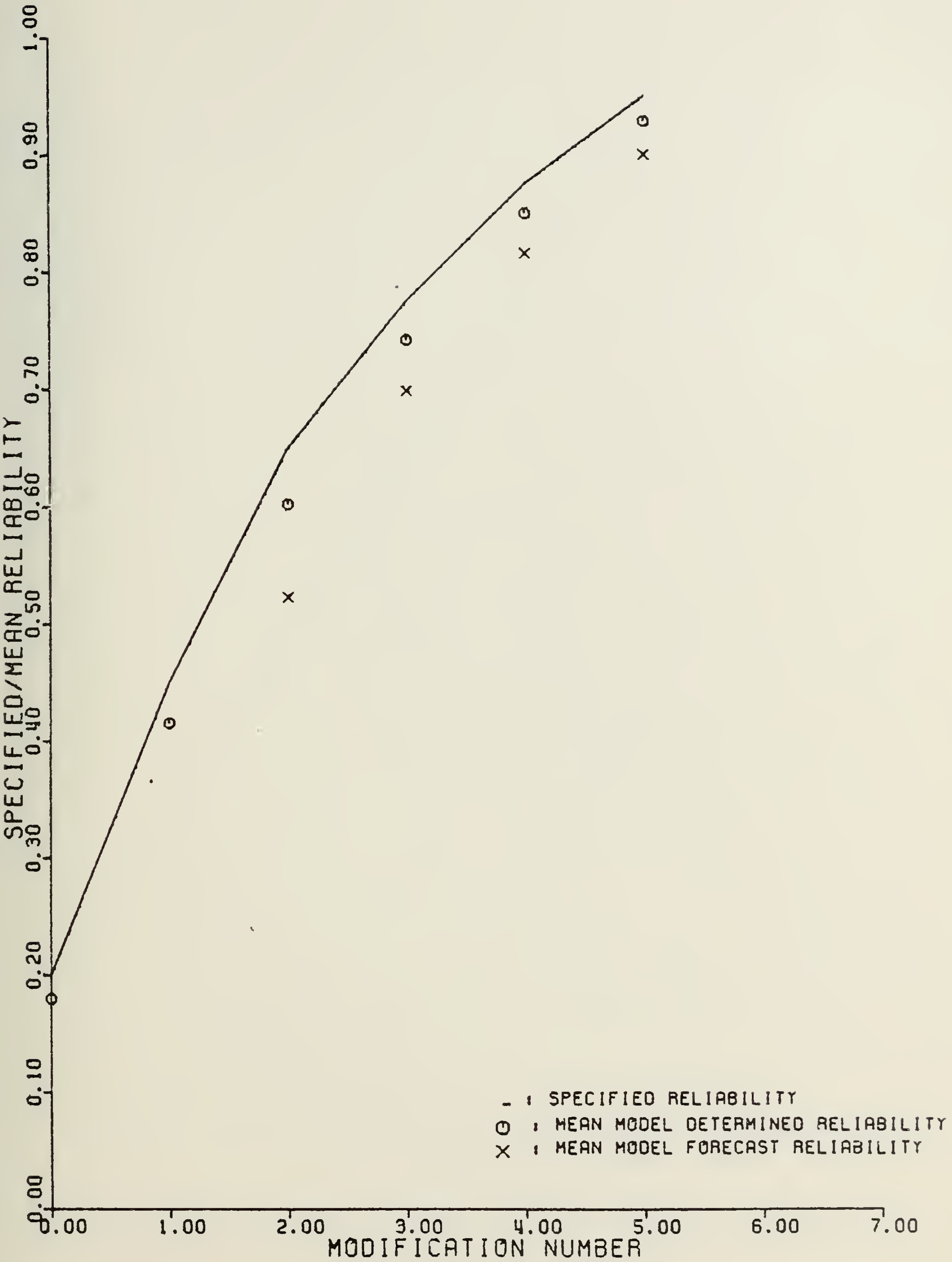

FIGURE 3.88

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 5: 5 MODS, 5 FAILURES/MOD

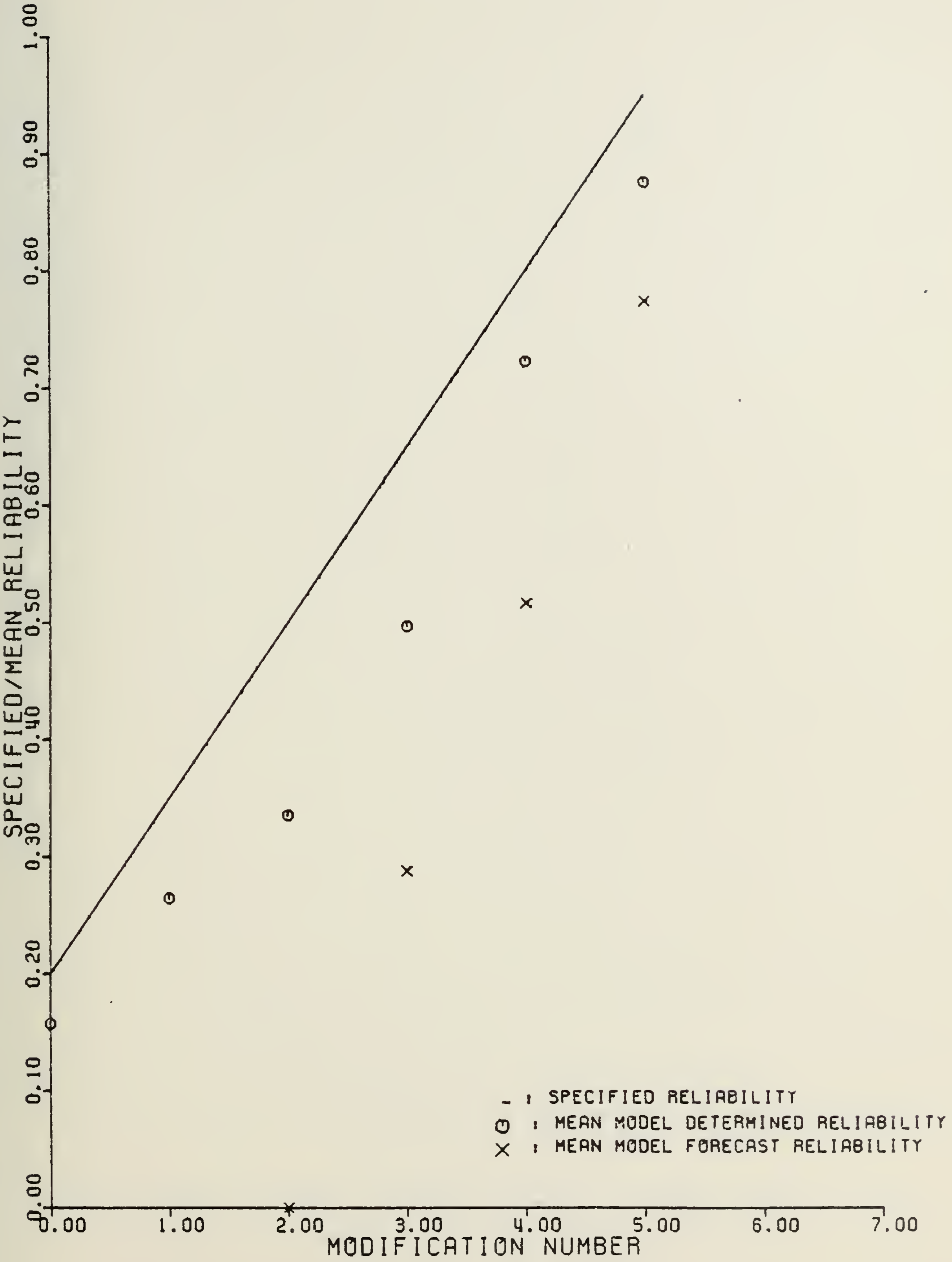

FIGURE 3.89

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 6: 5 MODS, 1 FAILURE/MOD

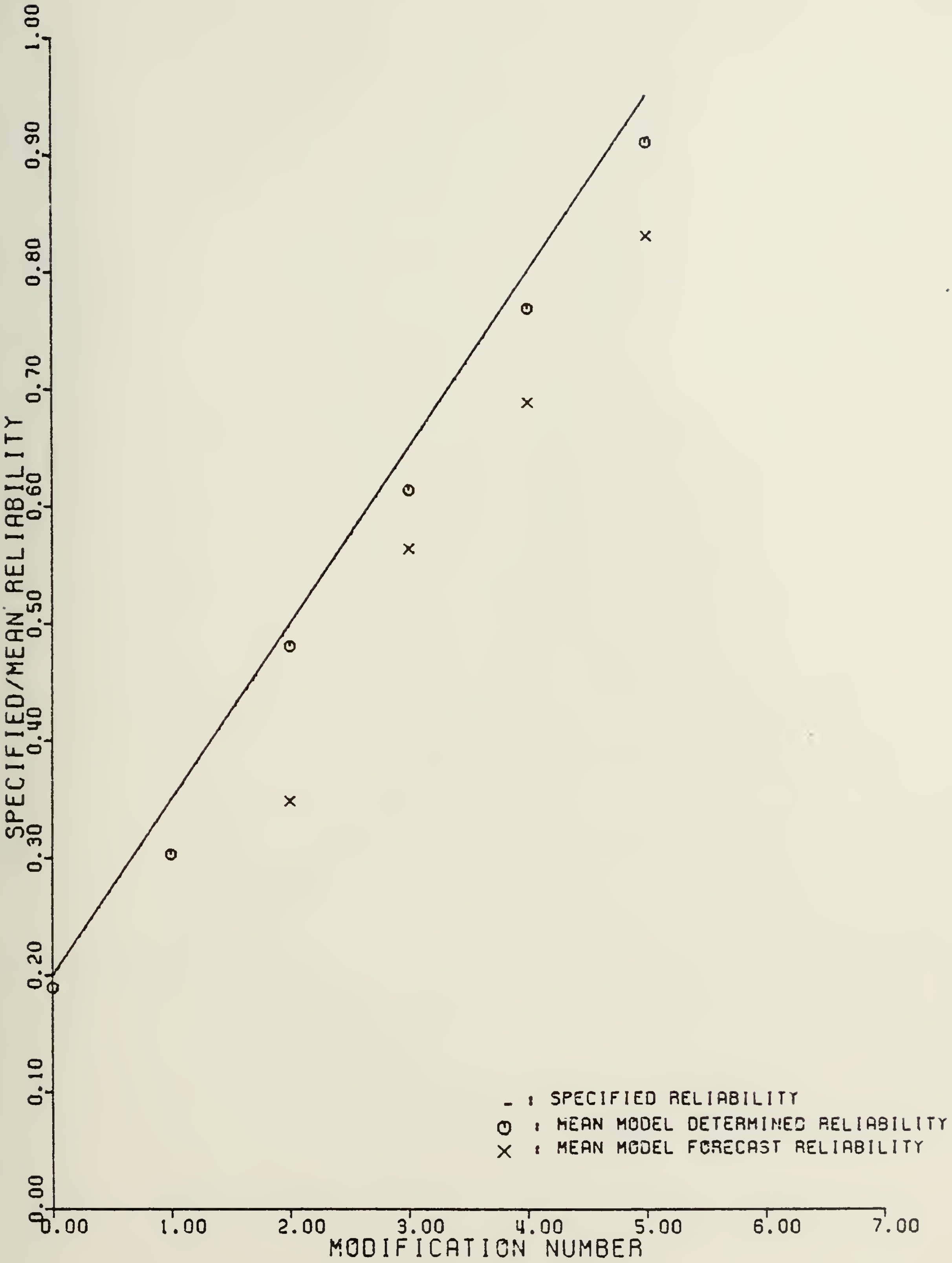

FIGURE 3.90
DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 6: 5 MODS, 5 FAILURES/MOD

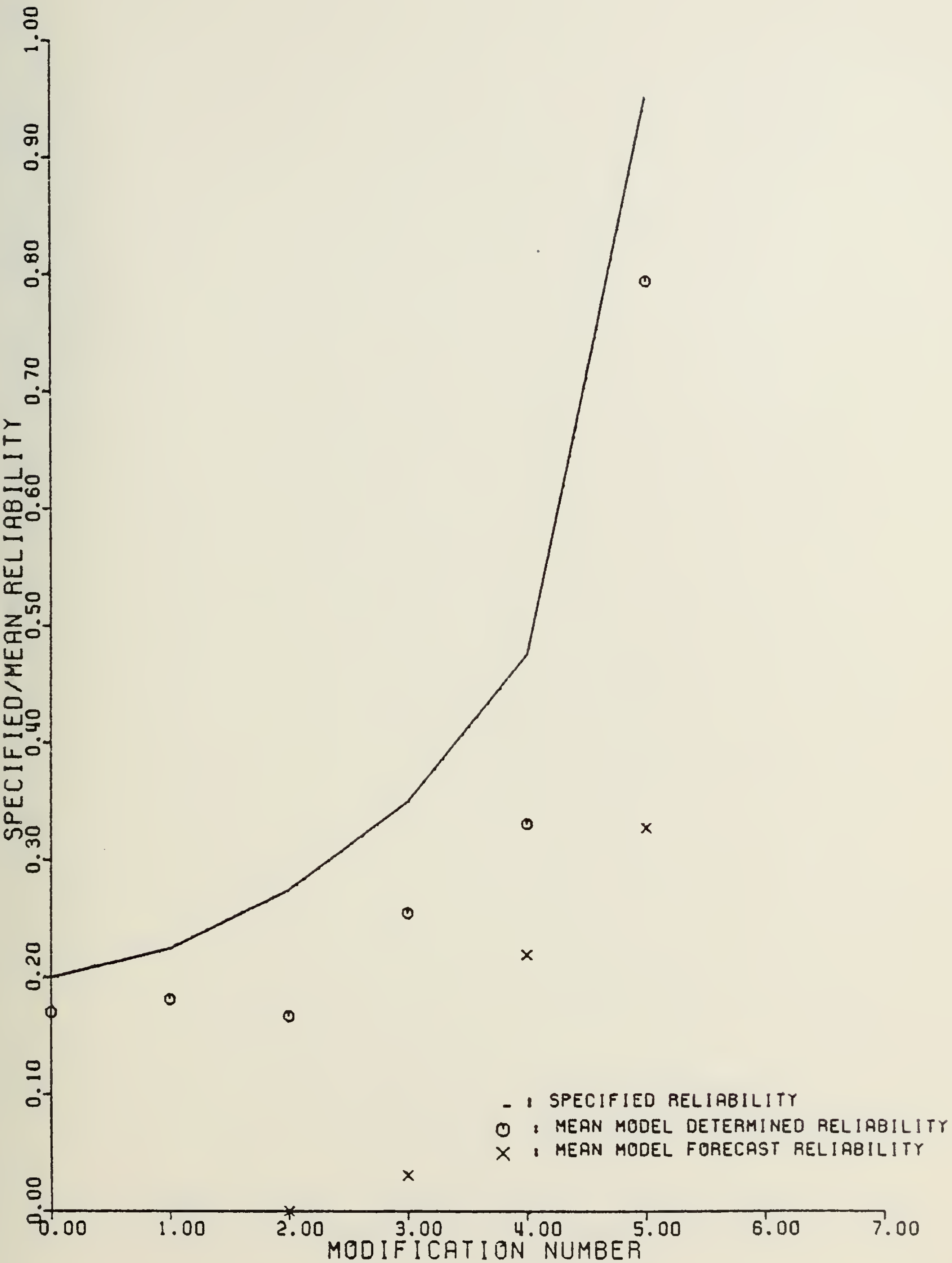

FIGURE 3.91
DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 7: 5 MODS, 1 FAILURE/MOD

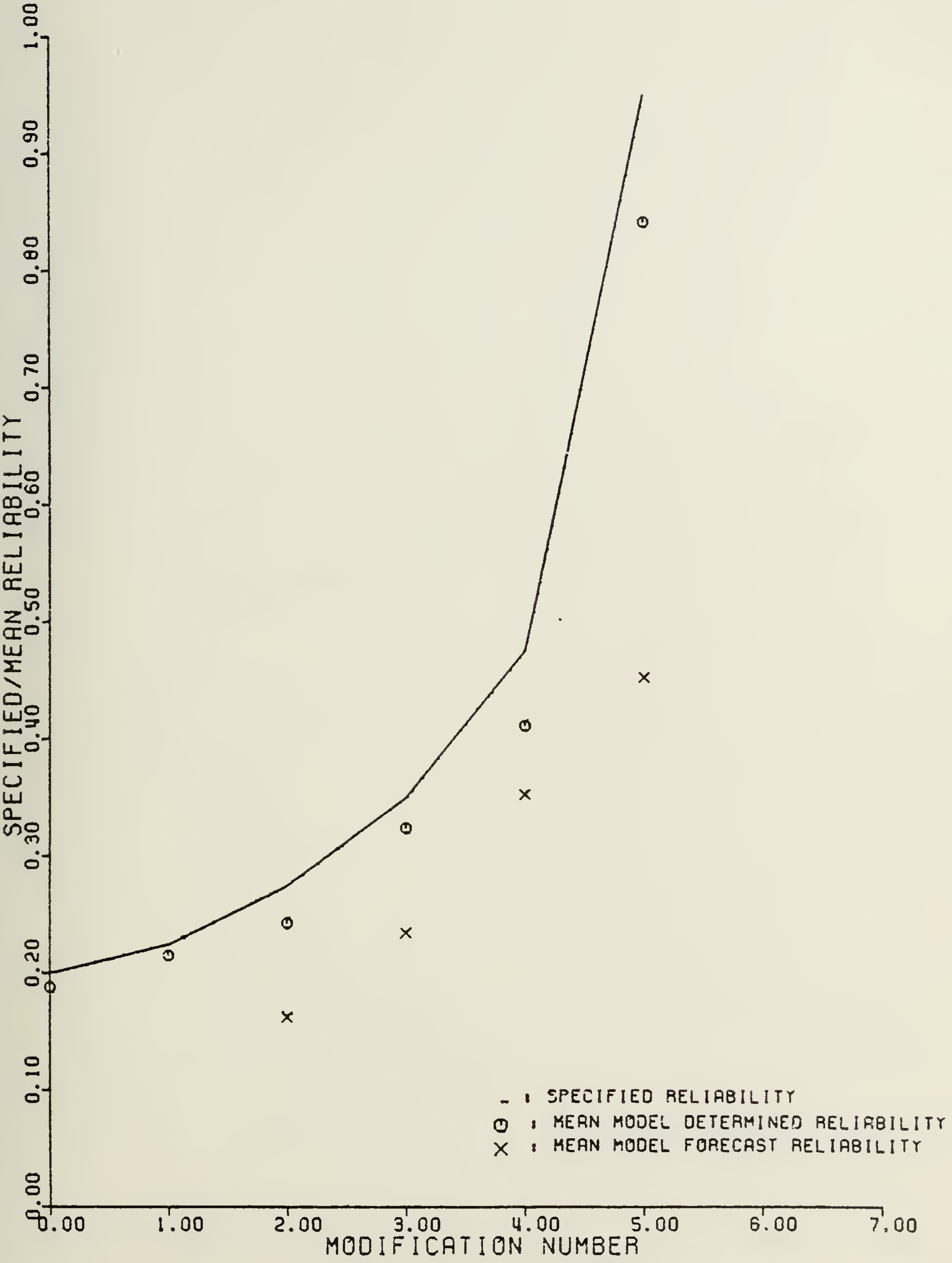

FIGURE 3.92

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 7: 5 MODS, 5 FAILURES/MOD

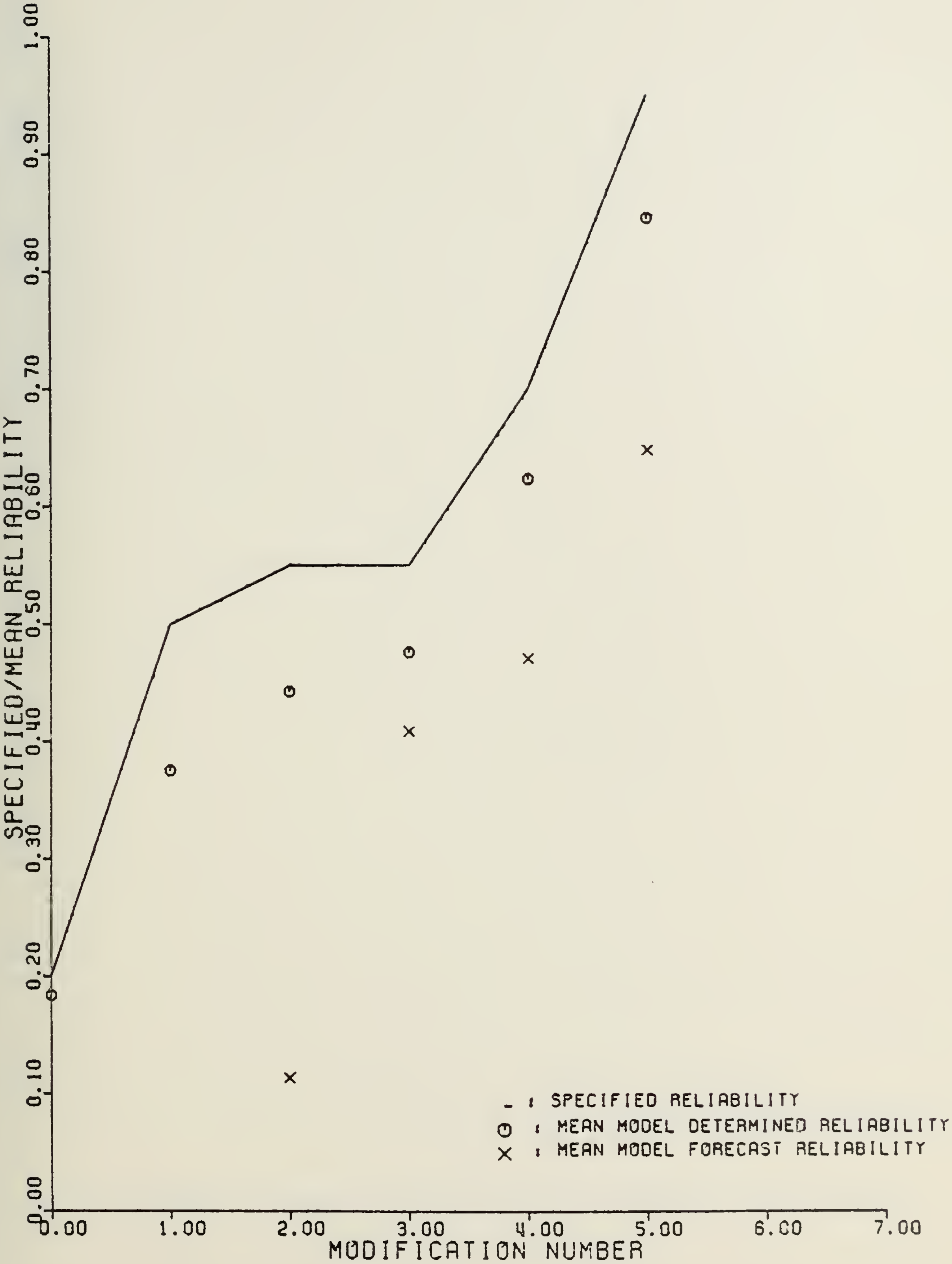

FIGURE 3.93

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 8: 5 MODS, 1 FAILURE/MOD

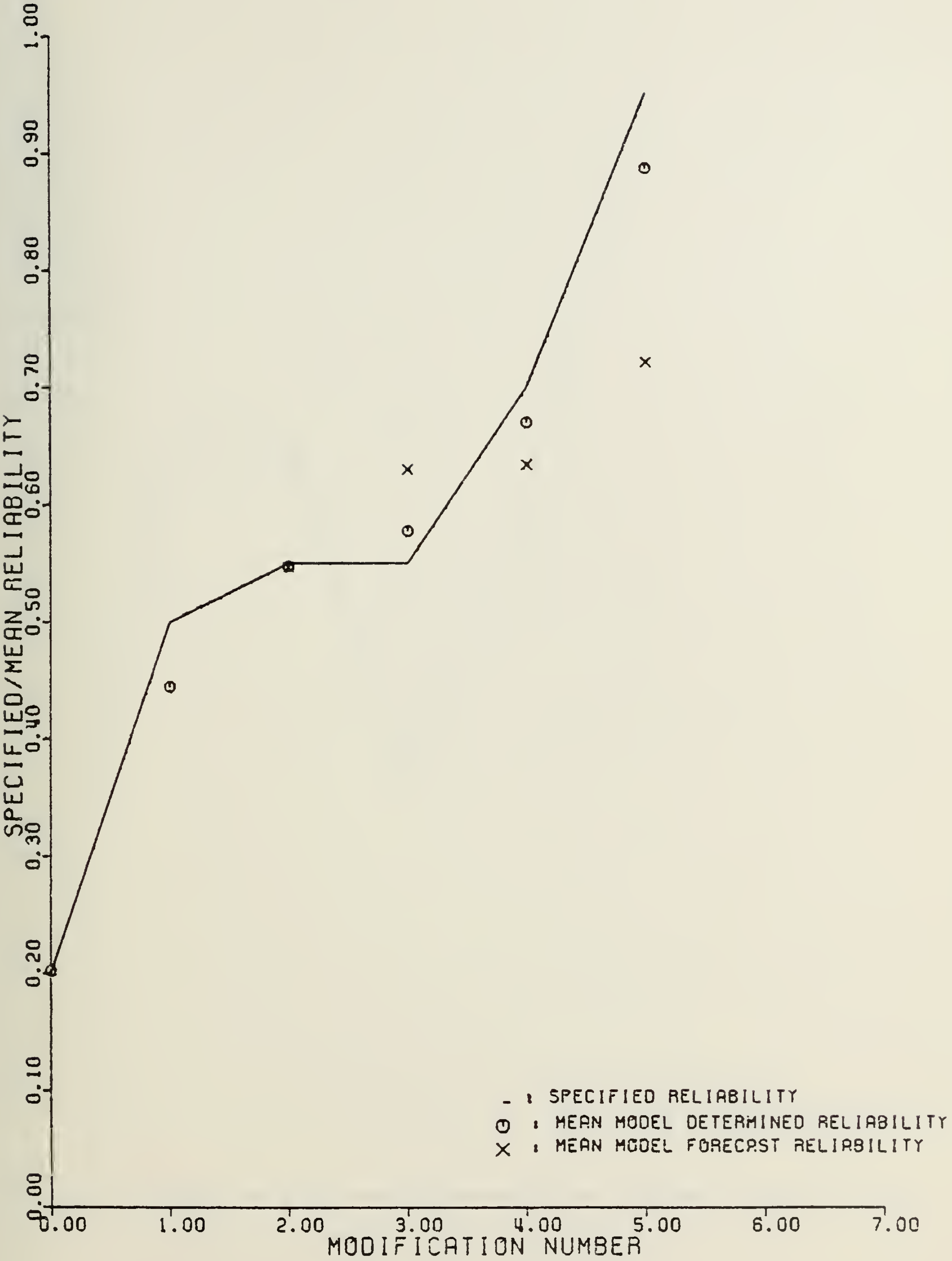

FIGURE 3.94

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 8: 5 MODS, 5 FAILURES/MOD

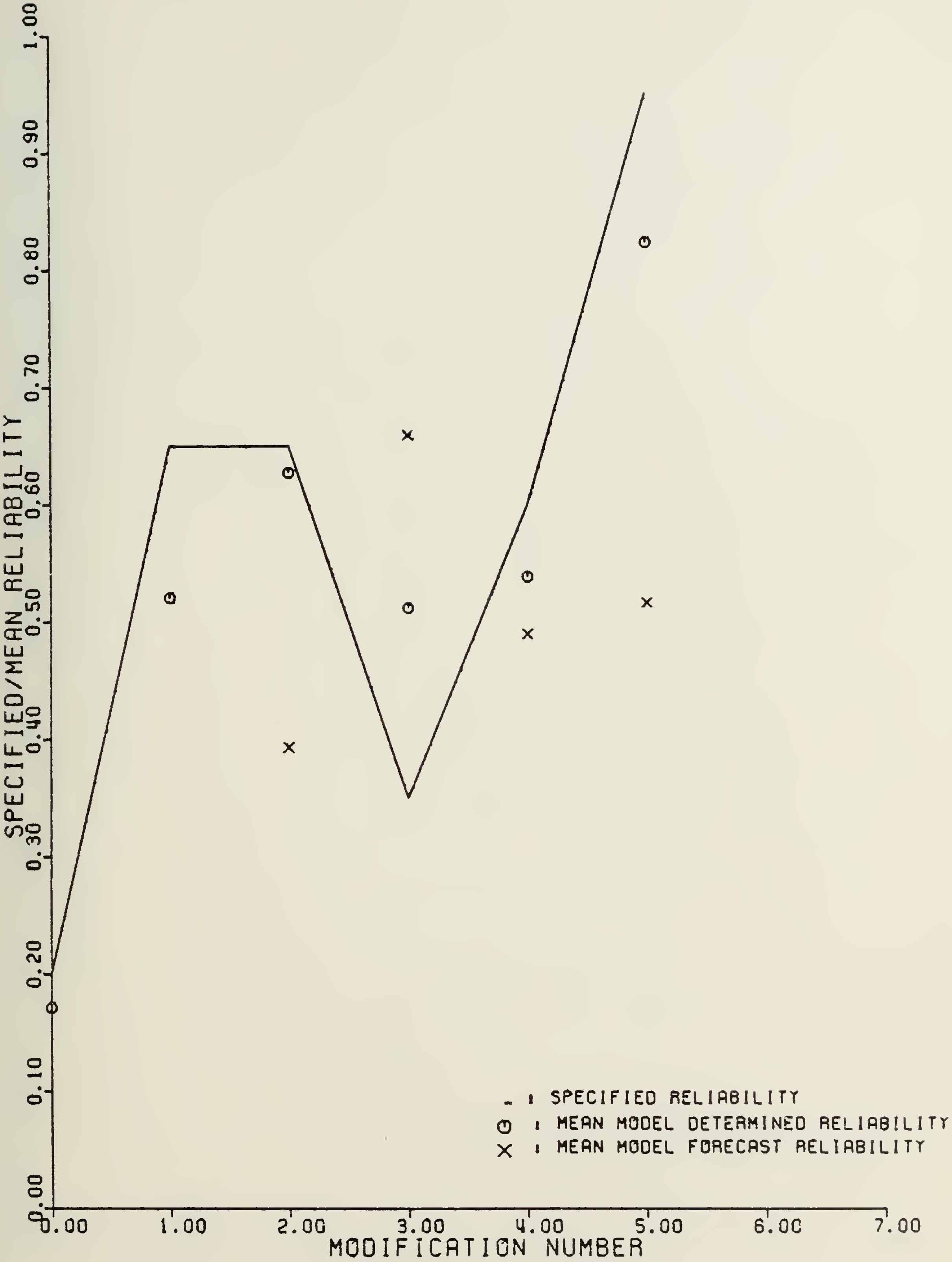

FIGURE 3.95
DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 9: 5 MODS, 1 FAILURE/MOD

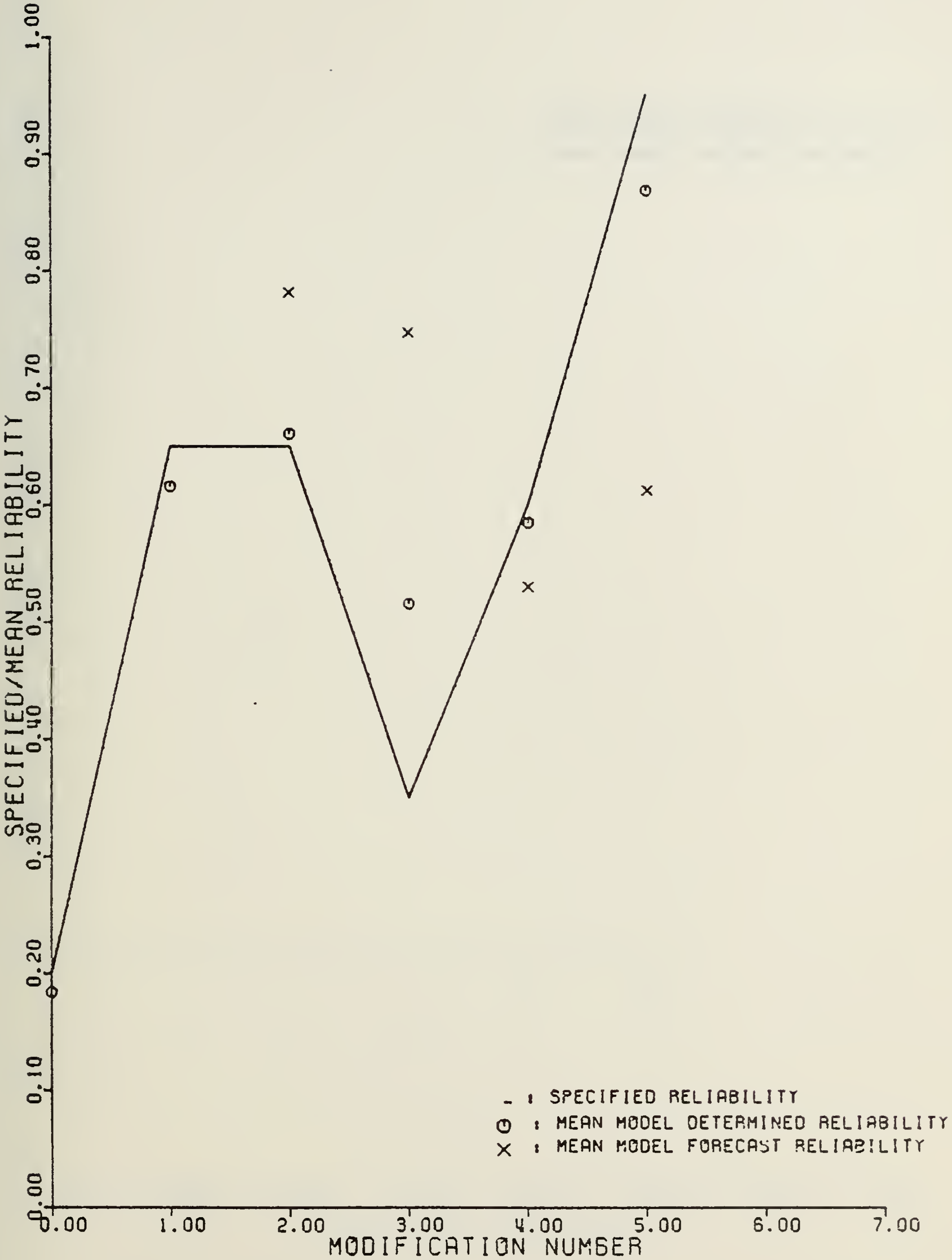

FIGURE 3.96

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 9: 5 MODS, 5 FAILURES/MOD

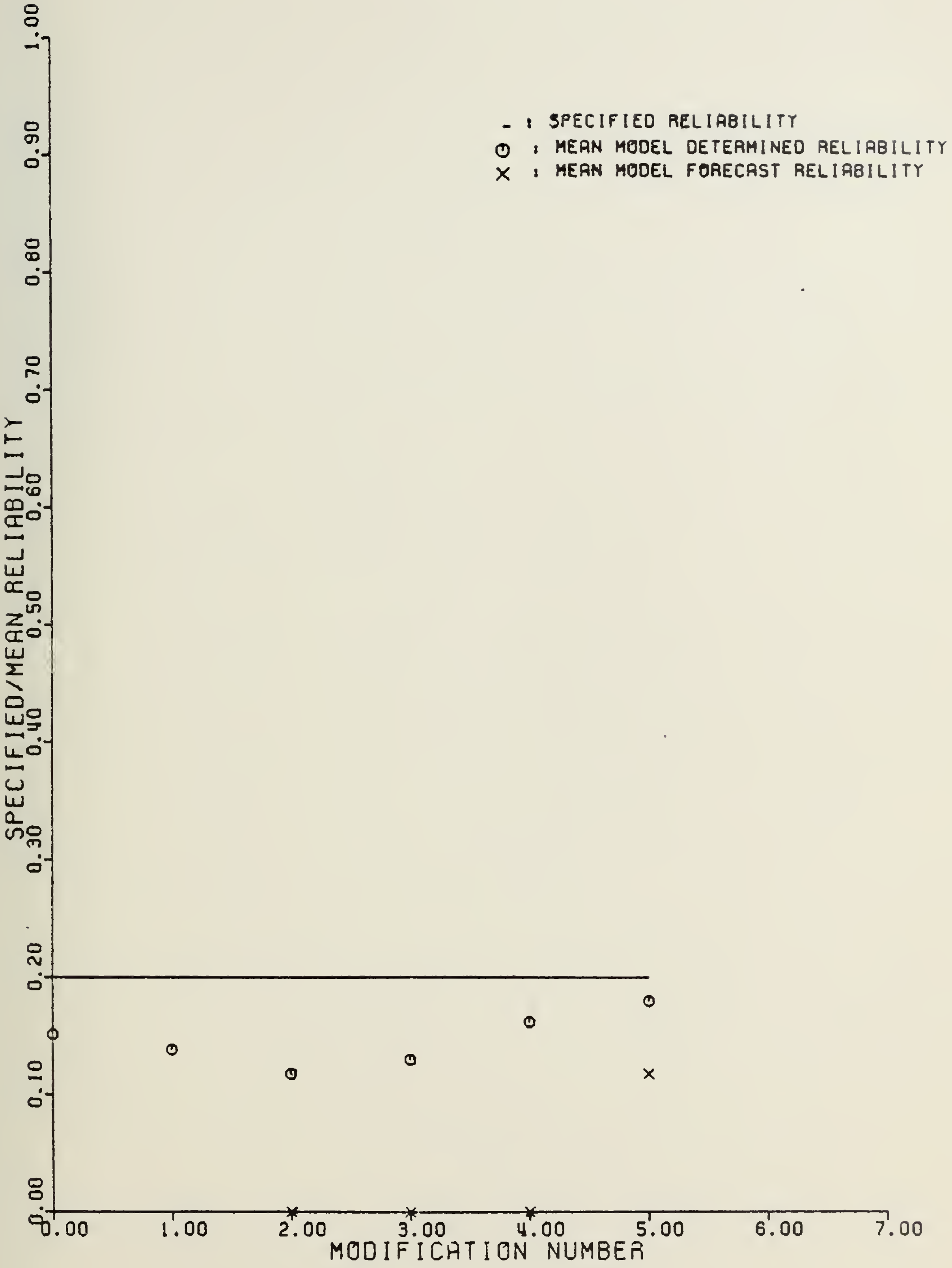

FIGURE 3.97

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 10: 5 MODS, 1 FAILURE/MOD

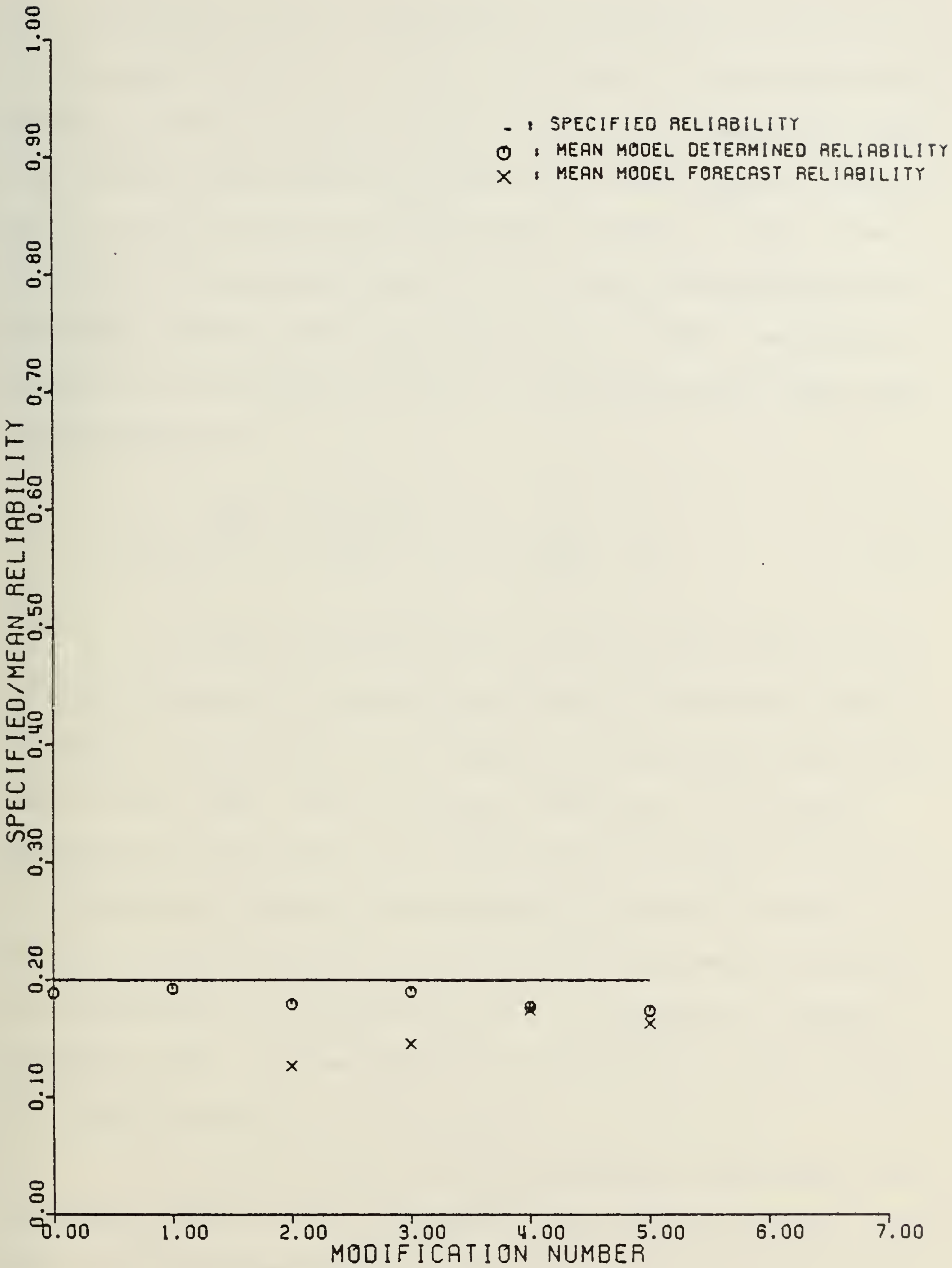

FIGURE 3.98

DISCRETE RELIABILITY GROWTH MODEL PERFORMANCE
RELIABILITY SET 10: 5 MODS, 5 FAILURES/MOD

the reliability status of a component as it undergoes modification in a system acquisition cycle. Entries in the tables are percentage standard errors as computed in equations 3.74 and 3.78. For example from figure 3.83 the mean model determined reliability for modification 3 from reliability set 3 was found to be $\overline{\widehat{R}}_3 = 0.86$ in the $NTF_3 = 1$ case. From table 3.28 the percentage standard error $P.S.E._{\widehat{R}_3}$ corresponding to this mean model determined modification reliability is 20%. Therefore, by equation 3.74 the standard deviation for the observed mean model determined reliability is

$$S.D._{\widehat{R}_3} = \overline{\widehat{R}}_3 \times \frac{P.S.E._{\widehat{R}_3}}{100} = 0.86 \times \frac{20}{100} = 0.172 \;. \qquad (3.79)$$

In tables 3.28, 3.29, and 3.30 the discrete model's variability performance for determining modification reliability status is displayed for the specified total number of failures until modification cases of one, three, and five failures, respectively. Two salient characteristics displayed by the results are (1) the large variability of the estimates produced by the reliability estimator of equation 3.71 and (2) the large variability of the reliability estimates produced by the discrete model for reliability set 10, the permanently stagnated reliability case. The same difficulty is displayed to a lesser degree for reliability set 7.

If reliability set 10 is not considered, then a goal of 40% percentage standard error is achieved by the discrete reliability growth model on the fifth modification for the $NTF_m = 1$ and 3 cases and on the fourth modification for the $NTF_m = 5$ cases. If reliability set 7 is also dropped from consideration, the 40% goal is achieved on the fifth

modification for $NTF_m = 1$, on the third modification for $NTF_m = 3$, and on the second modification for $NTF_m = 5$. Note that the reliability sets characterizing temporary reliability stagnation and degradation (sets 8 and 9) did not have to be taken out of consideration in order to find variability performance at a reasonable level.

Tables 3.31, 3.32, and 3.33 present the discrete model's variability performance in forecasting the reliability of the subsequent modification version of the component under test for the $NTF_m = 1$, 3, and 5 cases. Again, the worst variability performance occurs for reliability sets 7 and 10. Dropping reliability sets 7 and 10 from consideration, a 40% percentage standard error goal is achieved only in the case of $NTF_m = 5$ at the fourth modification. Hence, variability performance of the discrete reliability growth model for forecasting is not particularly good.

TABLE 3.28

DISCRETE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE
5 MODIFICATIONS, 1 FAILURE/MODIFICATION, DETERMINED RELIABILITY

	RELIABILITY SET									
MODIFICATION	1	2	3	4	5	6	7	8	9	10
0	231	240	170	195	201	179	170	161	170	184
1	32	38	64	100	104	126	166	98	69	195
2	12	24	34	53	64	99	170	76	47	214
3	8	15	20	26	32	62	116	62	49	213
4	4	9	13	16	18	27	84	38	48	172
5	4	5	7	6	8	11	19	16	21	137

TABLE 3.29

DISCRETE RELIABILITY GRCWTH MODEL VARIABILITY PERFCRMANCE

5 MODIFICATIONS, 3 FAILURES/MODIFICATION, DETERMINED RELIABILITY

	RELIABILITY SET									
MODIFICATION	1	2	3	4	5	6	7	8	9	10
0	105	108	107	117	128	119	104	116	110	109
1	11	14	28	45	53	70	118	46	30	109
2	2	10	14	17	28	41	91	33	22	116
3	1	4	5	9	16	31	64	28	27	106
4	1	2	4	5	7	12	41	18	30	110
5	1	1	2	2	3	5	8	5	8	93

TABLE 3.30

DISCRETE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE
5 MODIFICATIONS, 5 FAILURES/MODIFICATION, DETERMINED RELIABILITY

	RELIABILITY SET									
MODIFICATION	1	2	3	4	5	6	7	8	9	10
0	79	73	73	71	86	75	87	73	80	82
1	5	10	22	34	45	58	69	42	22	79
2	2	4	11	12	26	36	65	26	18	81
3	1	3	5	6	13	23	42	21	21	75
4	1	1	3	3	6	11	33	17	17	70
5	1	1	1	2	3	3	6	4	4	65

TABLE 3.31

DISCRETE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE
5 MODIFICATIONS, 1 FAILURE/MODIFICATION, FORECAST RELIABILITY

	RELIABILITY SET									
MODIFICATION	1	2	3	4	5	6	7	8	9	10
2	42	80	217	606	554	4004	593	957	244	484
3	11	41	57	77	84	192	1869	134	60	4475
4	8	20	25	28	36	78	190	84	73	920
5	4	10	16	16	19	27	112	46	70	343

TABLE 3.32

DISCRETE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE
5 MODIFICATIONS, 3 FAILURES/MODIFICATION, FORECAST RELIABILITY

	RELIABILITY SET									
MODIFICATION	1	2	3	4	5	6	7	8	9	10
2	7	11	31	53	87	150	923	56	31	767
3	1	9	14	16	29	53	166	38	24	440
4	1	3	4	7	15	37	82	35	41	205
5	1	2	3	4	6	12	46	19	41	196

TABLE 3.33

DISCRETE RELIABILITY GROWTH MODEL VARIABILITY PERFORMANCE

5 MODIFICATIONS, 5 FAILURES/MODIFICATION, FORECAST RELIABILITY

	RELIABILITY SET									
MODIFICATION	1	2	3	4	5	6	7	8	9	10
2	1	6	20	44	60	92	250	59	24	300
3	1	2	8	10	28	43	121	30	20	177
4	1	2	4	5	12	24	54	24	29	122
5	1	1	2	3	5	10	37	18	20	105

IV. CONTINUOUS FAILURE RATE RELIABILITY GROWTH MODELS COMPARISON

Codier in reference 2 gives a derivation in which an equation of the form of the continuous instantaneous failure rate reliability growth model (equation 3.37) is obtained by differentiating the equation of the continuous cumulative failure rate reliability growth model (equation 3.1) with respect to the total accumulated test time TT after substitution of the cumulative failure rate estimator of equation 3.13. The derivation essentially proceeds along the following line:

$$\lambda_{TT} \equiv \frac{TF}{TT} = \beta TT^{-\alpha} \; ; \quad \text{therefore,} \tag{4.1}$$

$$TF = \beta TT^{(1-\alpha)} \; . \tag{4.2}$$

Now,

$$\frac{\partial TF}{\partial TT} = \frac{\partial}{\partial TT} \left(\beta TT^{(1-\alpha)}\right) = (1-\alpha)\beta TT^{-\alpha} \; , \text{ and} \tag{4.3}$$

$$\lambda_T \equiv \frac{\partial TF}{\partial TT} \; . \tag{4.4}$$

Hence Codier derives an equation

$$\lambda_T = (1-\alpha)\beta TT^{-\alpha} \tag{4.5}$$

and claims that the equation is the continuous instantaneous failure rate model

$$\lambda_T = (1-a) \; b \; TT^{-a} \; ; \text{ i.e.,} \tag{4.6}$$

equation 3.37. The implication being that the cumulative model parameters α and β are equivalent to the instantaneous model parameters a and b , respectively; and so, once either α and β or a and b have been estimated, it is unnecessary to estimate the other parameter pair.

The cumulative and instantaneous failure rate models were considered as two unrelated models which were proposed to model different aspects of reliability growth from the viewpoint of this thesis. The evaluation of the two models was conducted accordingly. In order to investigate the proposition that the cumulative and instantaneous models are related thru equivalence of model parameters a hybrid cumulative failure rate model was devised that utilized the parameter estimates generated for the instantaneous failure rate model. Also, a hybrid instantaneous failure rate model was devised that utilized the parameter estimates generated for the cumulative failure rate model. Specifically, the hybrid cumulative failure rate model is given by

$$\widehat{\lambda^{*}_{TT_{i,r}}} = \hat{b}_{i,r} \, TT_{i,r}^{-\hat{a}_{i,r}} \qquad (4.7)$$

where $\hat{a}_{i,r}$ and $\hat{b}_{i,r}$ were determined by the instantaneous failure rate model equations 3.42 and 3.43. The hybrid instantaneous failure rate model is given by

$$\widehat{\lambda_{T_{i,r}}} = (1 - \hat{\alpha}_{i,r}) \, \hat{\beta}_{i,r} \, TT_{i,r}^{-\hat{\alpha}_{i,r}} \qquad (4.8)$$

where $\hat{\alpha}_{i,r}$ and $\hat{\beta}_{i,r}$ were determined by the cumulative failure rate model equations 3.21 and 3.22 after appropriate transformations given in chapter III.A.4.

As cumulative and instantaneous failure rate model parameter estimates were determined for each phase of each simulation for the different lambda sets they were applied to the hybrid models as well as to their appropriate models. Hybrid model estimates of the cumulative and instantaneous failure rates were recorded and the standard statistics of chapter III were compiled.

Table 4.1 contains the mean cumulative and instantaneous determined failure rate estimates produced by the hybrid models for the lambda set 3, ten tests per phase ($NT_i = 10$) case listed with the appropriate underlying mean test time weighted average cumulative failure rates $\overline{\lambda}_{TT_i}$ (equation 3.30) and the specified underlying instantaneous failure rates λ_i (table 3.1). Lambda set 3 is one of the "nice" failure rate sets for which both the cumulative and instantaneous failure rate models displayed some of their best accuracy and variability performance (see figures 3.5, 3.6, 3.7, and 3.44; and tables 3.4 and 3.16). Entries of 10.0000 for the hybrid cumulative model mean determined cumulative failure rate in table 4.1 indicate that the actual mean estimate was greater than or equal to 10.0000.

The performance of the hybrid models displayed in table 4.1 for a "nice" lambda set is typical of the behavior of the hybrid models for all the lambda sets, nice or anomalous. The hybrid cumulative model (instantaneous model's estimated parameters substituted into the cumulative model) produced failure rate estimates of magnitude greater than 10.0 for the first few test phases followed by very erratic estimates (including negative mean estimates) which displayed no relation to the mean test time weighted average cumulative failure rates supposedly being modelled. On the other hand the hybrid instantaneous model (cumulative

TABLE 4.1

HYBRID CUMULATIVE AND INSTANTANEOUS FAILURE RATE MODEL SIMULATION RESULTS LAMBDA SET 3: 16 PHASES, 10 TESTS/PHASE

PHASE *****	MEAN TEST TIME WEIGHTED AVERAGE CUMULATIVE FAILURE RATE **********	HYBRID CUMULATIVE MODEL MEAN DETERMINED CUMULATIVE FAILURE RATE **********	SPECIFIED INSTANTANEOUS FAILURE RATE *************	HYBRID INSTANTANEOUS MODEL MEAN DETERMINED INSTANTANEOUS FAILURE RATE *************
1	0.7000	0.6702	0.7000	0.6702
2	0.4703	10.0000	0.3530	0.3605
3	0.3565	10.0000	0.2380	0.3048
4	0.2863	10.0000	0.1800	0.2669
5	0.2392	10.0000	0.1450	0.2362
6	0.2064	-0.1003	0.1220	0.2173
7	0.1818	1.9321	0.1060	0.2016
8	0.1625	-0.0171	0.0933	0.1889
9	0.1474	-0.0065	0.0837	0.1796
10	0.1346	0.0532	0.0760	0.1684
11	0.1241	-0.0206	0.0697	0.1626
12	0.1152	0.0237	0.0644	0.1548
13	0.1076	0.0070	0.0600	0.1497
14	0.1009	0.2491	0.0562	0.1442
15	0.0952	0.0518	0.0529	0.1402
16	0.0900	0.0060	0.0500	0.1357

model's estimated parameters substituted into the instantaneous model) produced what appeared to be a good failure rate estimate for the first one or two phases ($i = 2, 3$; models do not estimate for the first phase) but quickly transitioned to a dampened, sedate pattern of estimates which was very unresponseive to even the most radical variations in the underlying instantaneous failure rate progress path. Also, the hybrid instantaneous model's failure rate estimates after the second or third phase generally exhibited increasing magnitude error except for crossing situations in the cases of temporary stagnation or degradation of the specified underlying instantaneous failure rates.

Regarding the negative failure rate estimates produced by the hybrid cumulative model (instantaneous model parameters in cumulative model), the following observations on the parameter estimates $\widehat{a_{i,r}}$ and $\widehat{b_{i,r}}$ of the continuous instantaneous failure rate model were made during the reliability testing procedure computer simulations. During the simulations the magnitude of the parameter estimate $\widehat{a_{i,r}}$ was continually very close to 1.0 often being more or less than 1.0 only in the tenth, eleventh, or twelfth decimal place. From the instantaneous model equation 4.6 it can be seen that logically if the $\widehat{a_{i,r}}$ estimate is less than 1.0, the $\widehat{b_{i,r}}$ estimate should be positive; and, if the $\widehat{a_{i,r}}$ estimate is greater than 1.0, the $\widehat{b_{i,r}}$ estimate should be negative in order to produce positive estimates of the instantaneous failure rate. Exactly such behavior was observed during the computer simulations; as the magnitude of the $\widehat{a_{i,r}}$ estimate moved to either side of 1.0 the sign of the $\widehat{b_{i,r}}$ estimate would "flip-flop" accordingly. Of course if a negative $\widehat{b_{i,r}}$ estimate is plugged into the cumulative failure rate model for β in equation 4.1, a negative estimate of the cumulative failure

rate necessarily results no matter what the sign or magnitude of the $\widehat{a}_{i,r}$ estimate which is substituted for α (save a value of $\widehat{a}_{i,r} = \infty$). Hence, it is very plausible for the hybrid cumulative model to have produced negative estimates of cumulative failure rate.

In contrast to the hybrid cumulative model's erratic failure rate estimates the hybrid instantaneous model (cumulative model parameters in instantaneous model) produced failure rate estimates which, after one or two test phases, displayed a "sluggish" growth pattern and generally increased in magnitude error from the underlying instantaneous failure rate λ_i being modelled. This situation is not surprising because the cumulative model parameters being employed in the hybrid instantaneous model are parameters utilized in modelling a "smoothed" quantity, the underlying cumulative (average) failure rate that is collectively characteristic of all the versions of an item tested thru a given point in the acquisition cycle. As the history of the item stretches farther and farther (total accumulated test time TT increases) the cumulative failure rate of the item becomes increasingly insensitive to the current instantaneous failure rate intrinsic to the item. Hence, the parameters of the cumulative model likewise become increasingly insensitive to the current instantaneous reliability status. When these relatively stable parameters are utilized in another model whatever its form, "smoothed" stable output is reasonably expected.

V. EVALUATION SUMMARY AND CONCLUSIONS

This thesis examined three reliability growth models for which accuracy, precision, and robustness characteristics were investigated over a wide variety of true underlying growth patterns, both "nice" and anomalous. The evaluation of a continuous cumulative failure rate model, an instantaneous failure rate model, and discrete reliability model was accomplished by computer simulation of reliability testing procedures that were appropriate to the reliability growth model types for a systems acquisition cycle. Performance of the models both in tracking and predicting true underlying reliability progress patterns was measured for mean accuracy and variability (precision) over progress patterns which included most situations that can be encountered, both good and bad. The models' performance characteristics are summarized in the following paragraphs.

A. CONTINUOUS CUMULATIVE FAILURE RATE RELIABILITY GROWTH MODEL

The continuous cumulative failure rate model examined generally displayed very good to excellent accuracy performance in tracking both "nice" and anomalous true underlying cumulative failure rate patterns. The cumulative model did exhibit difficulty in determining and forecasting true underlying patterns that characterize rapid reliability growth (failure rate decay) in the latter phases of the systems acquisition cycle. The difficulty was evidenced by mean model determined/forecast cumulative failure rate estimates that diverged from the true underlying values on the pessimistic side; thus, the cumulative model provided estimates that could be considered as upper bounds on failure rate in this

situation. The other anomalous situation with which the cumulative model experienced difficulty in tracking and predicting accuracy was one of reliability growth interrupted by a period of reliability degradation. The modelled cumulative failure rate path both lagged the true underlying cumulative failure rate path and failed to reflect the full magnitude of the failure rate degradation. Thus dependence on the model in this situation could hamper response to the degradation by giving a "too little too late" signal of the problem. The cumulative model coped with true underlying cumulative failure rate patterns that were permanently stagnated quite adequately.

As the reliability testing procedures of the acquisition cycles completed more test phases, the cumulative model's variability (precision) performance improved uniformly for both "nice" and anomalous true underlying failure rate paths. Variability percentage standard error goals were satisfied earlier and earlier in the acquisition cycle as more testing data was made available to the model. This good variability performance provides a degree of confidence in employing the cumulative failure rate model in actual systems acquisition programs where the intrinsic cumulative failure rate status is truly unknown.

B. CONTINUOUS INSTANTANEOUS RAILURE RATE RELIABILITY GROWTH MODEL

The continuous instantaneous failure rate model examined, while exhibiting the capability to track and predict the shape of various unknown, underlying instantaneous failure rate path types with very good accuracy and robustness, consistently produced optimistically biased estimates of the instantaneous failure rate. If bias must be accepted in a reliability growth model, then pessimistic bias is preferred to

optimistic bias. Consistent pessimistic bias permits managers to utilize the model produced estimates as lower bound type values and have confidence that reliability status is being observed from the "right side of the fence". On the other hand consistently optimistic model produced estimates impart a great deal of uncertainty as to how bad reliability status might be and what degree of corrective action is required.

The anomalous situations in which the instantaneous model experienced difficulty in tracking and predicting the shape of the true failure rate path were (1) the initial period of recovery in reliability growth after a period of temporary reliability stagnation, (2) permanently stagnated reliability growth at high failure rates, and (3) reliability growth interrupted by a temporary period of reliability degradation. In the case of temporary reliability degradation the instantaneous model charted a determined failure rate path that was an exaggeration of the underlying instantaneous failure rate path; i.e., the periods of growth were displayed optimistically and the period of degradation was portrayed to a magnitude greater than it was in truth. The instantaneous model's forecasts, while accurately capturing the shape of the underlying progress path, were consistently optimistic during the period of degradation.

The instantaneous model's accuracy performance was generally better for contracted acquisition cycles and suffered when the reliability testing procedure was extended to encompass a large number of test phases. Because of the model's consistent optimistic bias, accuracy performance improved as the true underlying failure rate decreased thereby "sandwiching" the model estimates between the true failure rate and 0.0. Also, the instantaneous model routinely furnished "off-the-scale" forecasts for the first one or two phases of an acquisition cycle.

The instantaneous model's demonstrated capability to determine and forecast the true underlying reliability status was overshadowed by its

poor variability performance which (1) never achieved a really comfortable percentage standard error goal uniformly for all failure rate patterns simulated and, more damaging, (2) oscillated as test phases of the reliability testing procedure were accomplished. Because of this poor variability performance, the instantaneous failure rate model cannot be employed by itself with any degree of confidence.

C. DISCRETE RELIABILITY GROWTH MODEL

The discrete reliability growth model examined generally displayed very good determined reliability accuracy performance even for the most restricted test data cases. Slight pessimistic bias was demonstrated for model determined reliability status while a greater magnitude of pessimistic bias was present in model forecast reliability. Again, pessimistic bias is favored over optimistic bias in charting a reliability progress path. The degree of pessimism in determined and forecast reliability decreased markedly as more test data on each modification version of an item were gathered. In cases of limited test data the discrete model often forecast negative estimates of reliability for the early modification versions of a component under test.

For the anomalous situation of temporary reliability degradation the discrete model (1) failed to determine the full magnitude of the degradation and (2) provided reliability status predictions that lagged the true underlying reliability path outcomes significantly. In actual testing this performance characteristic would give a delayed signal of a situation that required corrective action. Also, gathering additional test data failed to remedy this deficiency in the discrete model's performance.

Variability performance of the discrete model, while generally good, revealed difficulty with the permanently stagnated reliability case and the case of rapidly increasing reliability during the latter modification versions of an item. Variability performance improved uniformly for all progress path characterizations as more modifications of the item being tested were accomplished. Also, variability goals were satisfied earlier in the acquisition cycle as more complete testing was accomplished on each modification version of the item. This nice behavior of the discrete model lends confidence to its utilization; and therefore, the discrete model is preferred to the continuous instantaneous failure rate model for obtaining a measure of the current or "instantaneous" reliability status of an item proceeding thru an acquisition cycle.

D. GENERAL OBSERVATIONS

For determining current reliability status it may be suggested that rather than employing the reliability growth models, why not utilize the point estimators of reliability status which were used to provide data to the models? Although the performance of the point estimators appropriate to each reliability growth model were only observed at one point for each model in the reliability testing procedure simulations (first phase of testing or mod 0 version of a component), at that point the estimators displayed very poor variability performance upon which the reliability growth models improved rapidly and significantly which tends to negate any confidence in utilizing the point estimators based on their good accuracy performance.

In retrospect the performance characteristics demonstrated by the reliability growth models examined suggest that as a model is employed the estimated reliability progress path it produces be compared with the

appropriate model accuracy performance graphs in chapter III. If the estimated reliability progress path corresponds to a case for which the computer simulation performance results show the model's performance was good, then confidence can be placed in the estimated progress path. On the other hand if the estimated path corresponds to one of the anomalous cases where simulation results revealed a deficiency in the model's performance, then the simulation performance results at least give an indication of which direction the true underlying reliability progress path lies.

Finally, although use of the continuous instantaneous failure rate reliability growth model is questionable, since the test data required for the instantaneous model is identical to the data collected for the continuous cumulative failure rate model, application of the instantaneous model simultaneously with the cumulative model utilizing the method described in the preceding paragraph may provide some insight to the shape of the true underlying instantaneous failure progress path from the instantaneous model. But the comparison performed between the two continuous failure rate models definitely indicates that the cumulative failure rate and instantaneous failure rate models' parameters not be interchanged based on their hypothetical equivalence.

APPENDIX A

Derivation of Ordinary Least Squares Regression Estimates of the Continuous Instantaneous Failure Rate Reliability Growth Model Parameters

The continuous instantaneous failure rate model reliability growth model as given in equation 3.37 may be written as

$$\lambda_i = b(1-a)TT_i^{-a} \quad . \tag{A.1}$$

Taking the logarithmic transformation of this equation yields

$$\ln \lambda_i = \ln b(1-a) - a \ln TT_i \quad . \tag{A.2}$$

Let

$$\sum = \sum_{i=1}^{n} \quad ,$$

$$Y_i = \ln \lambda_i \quad ,$$

$$X_i = \ln TT_i \quad ,$$

$$\bar{Y} = \frac{\sum Y_i}{n} \quad , \quad \text{and}$$

$$\bar{X} = \frac{\sum X_i}{n} \quad .$$

Therefore,

$$Y_i = \ln b(1-a) - aX_i \tag{A.3}$$

The residual error for regression is

$$\varepsilon_i = Y_i - \widehat{Y}_i = Y_i - [\ln b(1-a) - aX_i] \; . \qquad (A.4)$$

The goal of the regression is to minimize the sum of the residual errors squared; i.e.,

$$\min \sum \varepsilon_i^{\,2} = \sum (Y_i - \widehat{Y}_i)^2 = \sum [Y_i - \ln b(1-a) + aX_i]^2 . \qquad (A.5)$$

Taking partial derivatives of equation A.5 with respect to b and a ; then equating the partials to 0 yields the following two equations in two unknowns:

(1) $$\frac{\partial \sum \varepsilon_i^{\,2}}{\partial b} = \sum [2 \; (Y_i - \ln b(1-a) + aX_i)(- \frac{1}{b(1-a)} \; (1-a)] \quad = 0$$

(2) $$\frac{\partial \sum \varepsilon_i^{\,2}}{\partial a} = \sum [2 \; (Y_i - \ln b(1-a) + aX_i)(- \frac{1}{b(1-a)}(-b) + X_i)] = 0$$

Simplification of (1) and (2) yields in sequence:

(1') $$\sum [Y_i - \ln b(1-a) + aX_i] = 0 \; , \text{ and}$$

(2') $$\sum [(Y_i - \ln b(1-a) + aX_i) \; (\frac{1}{(1-a)} + X_i)] = 0 \; . \quad \text{Then,}$$

(1'') $$\sum Y_i = \sum \ln b(1-a) - \sum aX_i \; ,$$

(2'') $$\sum \frac{1}{(1-a)} Y_i + \sum Y_i X_i = \sum \frac{1}{(1-a)} \ln b(1-a) + \sum X_i \ln b(1-a) - \sum \frac{aX_i}{(1-a)} - \sum aX_i^{\,2} \; . \quad \text{Finally,}$$

$$(1''') \qquad \sum Y_i = n \ln b(1-a) - a \sum X_i \,, \quad \text{and}$$

$$(2''') \quad \frac{1}{(1-a)} \sum Y_i + \sum Y_i X_i = \frac{n}{(1-a)} \ln b(1-a) + (\ln b(1-a)) \sum X_i - \frac{a}{(1-a)} \sum X_i - a \sum X_i^2 \,.$$

Equation (1''') may be manipulated such that

$$(1'''') \quad \ln b(1-a) = \frac{1}{n} \sum Y_i + \frac{a}{n} \sum X_i = \bar{Y} + a\bar{X} \quad \text{or}$$

$$(1''''') \quad b(1-a) = \exp\,(\bar{Y} + a\bar{X}) \,.$$

Finally, the estimate for b is

$$\hat{b} = \frac{1}{(1-\hat{a})} \exp\,(\bar{Y} + \hat{a}\,\bar{X}) \,. \tag{A.6}$$

Substituting equation A.6 into (2''') yields

$$(2'''') \quad \frac{1}{(1-a)} \sum Y_i + \sum Y_i X_i = \frac{n}{(1-a)} \ln\left[\frac{\exp\,(\bar{Y} + a\bar{X})}{(1-a)}\,(1-a)\right] + \ln\left[\frac{\exp\,(\bar{Y} + a\bar{X})}{(1-a)}\,(1-a)\right] \sum X_i - \frac{a}{(1-a)} \sum X_i - a \sum X_i^2 \,.$$

Clearing the logarithms in (2'''') and transposing leaves

$$(2''''') \quad \frac{n}{(1-a)} (\bar{Y} + a\bar{X}) + (\bar{Y} + a\bar{X}) \sum X_i - \frac{a}{(1-a)} \sum X_i - a \sum X_i^2 - \frac{1}{(1-a)} \sum Y_i = \sum Y_i X_i \,.$$

Expanding (2''''') yields

$$(2^*) \quad \frac{n}{(1-a)} \bar{Y} + \frac{an}{(1-a)} \bar{X} + \bar{Y} \sum X_i + a\bar{X} \sum X_i - \frac{a}{(1-a)} \sum X_i - a \sum X_i^2 - \frac{1}{(1-a)} \sum Y_i = \sum Y_i X_i \,.$$

Substituting and clearing terms in (2*) leaves

$$(2^{**})\quad \frac{1}{(1-a)} \sum Y_i + \frac{a}{(1-a)} \sum X_i + \bar{Y} \sum X_i + a\bar{X} \sum X_i - \frac{a}{(1-a)} \sum X_i - a \sum X_i^2 - \frac{1}{(1-a)} \sum Y_i = \sum Y_i X_i .$$

Solving for a in (2**) gives

$$(2^{***})\quad a(\bar{X} \sum X_i - \sum X_i^2) = \sum Y_i X_i - \bar{Y} \sum X_i \quad \text{or}$$

$$\hat{a} = \frac{\sum Y_i X_i - \bar{Y} \sum X_i}{\bar{X} \sum X_i - \sum X_i^2} . \qquad \text{(A.7)}$$

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